BC5 : 2019

DESIGN GUIDE FOR THE USE OF GRADE B600 HIGH STRENGTH REINFORCEMENT IN REINFORCED CONCRETE STRUCTURES

Building and Construction Authority

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Foreword

The use of steel reinforcement with yield strength of 600 MPa is gaining popularity in Singapore. The use of Grade B600 reinforcement bars (rebars) in structural concrete offers several advantages. With a higher specified yield strength, f_y, the designer can reduce the cross-sectional area of required reinforcement, and save cost on material, shipping, and placement. The reduced area of reinforcement results in fewer bars and reduces reinforcement congestion often encountered in mat foundations, shear walls, beam-column joints, and many precast concrete elements. The reduction in reinforcement congestion facilitates concrete placement and consolidation, and leads to better quality construction, improved durability of the structure, and a reduction in construction time and cost.

However, there are also disadvantages in using Grade B600 rebars in structural concrete design. Using higher specified yield strength, f_y, may result in higher steel stress at service load condition and potentially cause wider cracks and larger deflections, which may be objectionable if aesthetics and water-tightness are critical design requirements. Also, with higher f_y, the required development length will be longer and may not be economical and practical for large size bars.

The current SS EN 1992-1-1 (EC2) code provisions for reinforced concrete (RC) structures limit the nominal yield stress, f_y , of longitudinal steel reinforcement to 600 MPa. The EC2 provides the principles and application rules on design but does not prescribe guidance or procedures to assist the designers. Unlike the common grade of steel of 500 MPa and below, there are also insufficient verifications of the design using the higher grade steel. The lack of information regarding the behaviour of concrete members reinforced with high-strength steel reinforcement hinders design engineers from using the full strength of the material.

This guide provides design provisions for the use of Grade B600 steel reinforcement for reinforced concrete structural members. This guide aims to address some situations in SS EN 1992-1-1 that require special deliberations or considerations. It also provides an overview of issues in the use of high strength steel reinforcement (having yield strength > 500 MPa). It includes recommendations on conformity compliance of connectors, ductility for moment redistribution, and use of confinement and/or time-dependant effects to increase the concrete strain for columns under pure compression loads. Examples are provided to illustrate design procedures and proper application of the recommendations. Modifications to these design recommendations may be justified where the design adequacy within the scope of this guide is demonstrated by successful use, analysis, or test.

While this document aims to provide a guidance and addition considerations on the use of high strength steel reinforcement in reinforced concrete construction, the design of concrete structures shall still fully comply with SS EN 1992-1-1.

Acknowledgement

The Building and Construction Authority of Singapore (BCA) would like to thank the authors for developing this Guidebook.

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1 General

Limits by design codes on maximum values of yield stress of longitudinal reinforcing bars permitted for structural design have changed over time because of advances in metallurgy, production of tougher reinforcing bars, desire for cost reduction, and availability of experimental data to support the changes. Steel reinforcement with yield strength of 600 MPa is commercially available in Singapore and there is even a local producer for this grade. Its use is expected to be more wide-spread in Singapore in the near future.

There are many potential benefits from the use of high-strength steel reinforcement in reinforced concrete construction in Singapore. These include less material, manpower reduction, reduced construction time, reduction of reinforcement congestion, lower cost etc. Such benefits serve to motivate the building authority to take the initiative to encourage the industry stake holders in using the high strength steel. A design guide culminating from this report is one of the initiatives.

The current SS EN 1992-1-1 (EC2) code provisions for reinforced concrete (RC) structures limit the nominal yield stress, f_y , of longitudinal steel reinforcement to 600 MPa. The EC2 provides the principles and application rules on design but does not prescribe guidance or procedures to assist the designers. Unlike the common grade of steel of 500 MPa and below, there are also insufficient verifications of the design using the higher grade steel. The lack of information regarding the behaviour of concrete members reinforced with high-strength steel reinforcement hinders design engineers from using the full strength of the material.

While this document aims to provide a guidance and addition considerations on the use of high strength steel reinforcement in reinforced concrete construction, the design of concrete structures shall still fully comply with SS EN 1992-1-1.

2 Material

The product specifications for reinforcement shall be based on the following:

- Rebar : SS 560 : 2016 - Steel for the reinforcement of concrete - Weldable reinforcing steel – Bar, coil and decoiled.
- Mesh : SS 561 : 2010 - Steel fabric for the reinforcement of concrete.

The Singapore Standard SS 560 : 2016 specifies requirements for ribbed weldable reinforcing steel used for the reinforcement of concrete structures. The standard covers steel delivered in the form of bars, coils and decoiled products. The weldability requirements for all grades of steel are specified in terms of the chemical composition, and in particular the carbon equivalent value. Steel bars produced by re-rolling finished products, or by rolling material of which the metallurgical history is not fully documented or not known, are not covered by this Singapore Standard.

SS 560 : 2010 was revised to include Grade B600 steel. This revised standard is an adoption of BS 4449 + A2 : 2009 'Steel for the reinforcement of concrete - Weldable reinforcing steel- Bar, coil and decoiled product - Specification' and is implemented with the permission of the British Standards Limited. The current standard SS 560:2016 contains provisions for both of 500 MPa and 600 MPa characteristic yield strength, but with different ductility characteristics. This standard has been written so that it can be used in conjunction with BS EN 10080 : 2005. BS EN 10080 : 2005 does not define steel grades or technical classes, and requires that technical classes should be defined in accordance with BS EN 10080 : 2005, by specified values of R_e, R_m/R_e, A_{gt}, $R_{e,act}/R_{e,nom}$ (where appropriate), fatigue strength, bend performance, weldability, bond strength, tolerances and dimensions.

It should be noted in accordance with SS 560 : 2016, purchasers specify reinforcing steel that has been manufactured and supplied, to conform with "Evaluation of Conformity" (Clause 8) through a recognised third party product certification scheme. As an alternative, Annex B provides a batch testing method for material which has not been produced under such a scheme.

3 Bar continuity and termination

For almost 100 years, construction practices in the building of concrete structures have focused on the use of steel reinforcement to transfer tension and shear forces. Lap splicing has become the traditional method of connecting the steel reinforcing bars, largely due to a misconception that lap splicing is “no-cost” splicing. Lap splicing requires the overlapping of two parallel bars. The overlap load transfer mechanism takes advantage of the bond between the steel and the concrete to transfer the load. The load in one bar is transferred to the concrete, and then from the concrete to the ongoing bar. The use of laps can be time consuming in terms of design and installation and can lead to greater congestion within the concrete because of the increased amount of rebar used. Lapped joints are also dependent upon the concrete for load transfer. For this reason, any degradation in the integrity of the concrete could significantly affect the performance of the joint.

Reinforcing bar couplers available in the market have come across with a solution for this complexity as it provides a greater ease in design and construction of reinforced concrete and reduce the amount of reinforcement required. The strength of a mechanical splice is independent of the concrete in which it is located and will retain its strength despite loss of cover as a result of impact damage or seismic event. Different types of couplers are shown in Figure 3-1 :

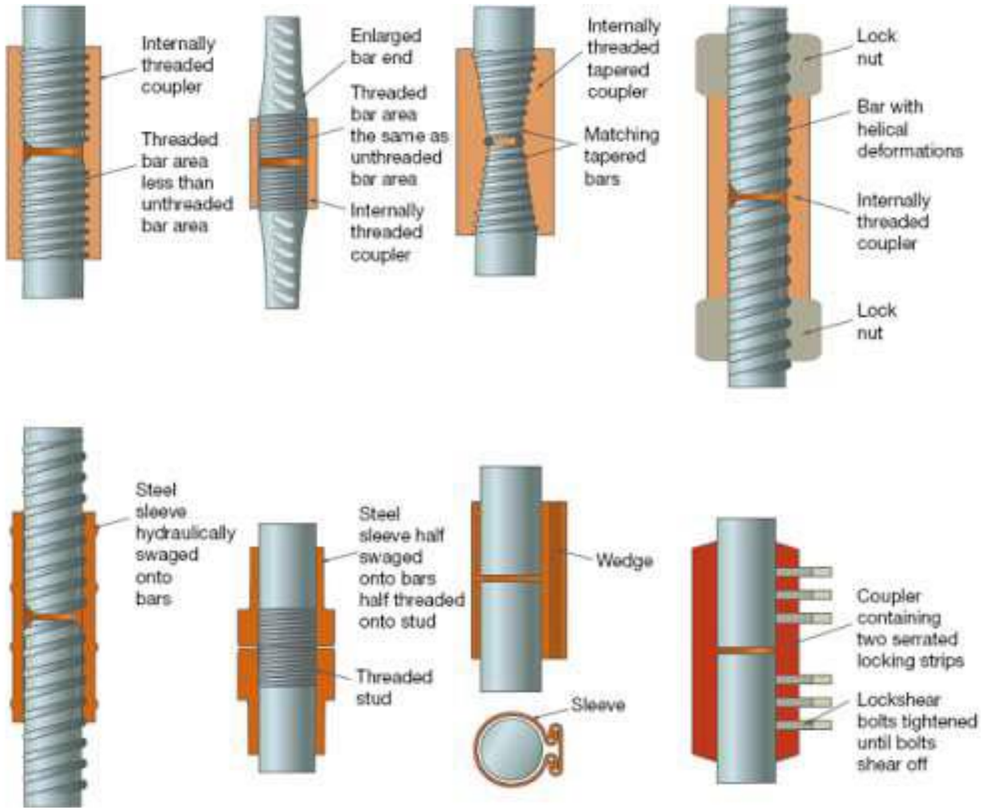


Figure 3-1 Different types of couplers
(https://www.strukts.com/blog/2091/mechanical-couplers-for-bars/)

In EC2, the requirements on rebar lapping are covered in clause 8.7. However, while mechanical devices are allowed to transfer load, there is no guidance or requirements on the use of couplers. Hence, like the steel reinforcement, a product standard is required to ensure good and proper use of couplers. The standard ISO 15835 shall be adopted for that purpose. This standard specifies requirements for couplers to be used for mechanical splices in reinforced concrete structures under predominantly static loads. It specifies additional requirements for couplers to be used in structures subject to high-cycle elastic fatigue loading and/or to low-cycle elastic-plastic reverse loading. The Conformity assessment scheme in ISO 15835, similar to that specified in SS 560 : 2016 for the rebars, provides the rules for the certification and for the self-evaluation of couplers to be used for the mechanical splicing of steel reinforcing bars. It also includes requirements for the control of the manufacturing process of the couplers and for the verification of their conformity.

The ISO 15835: 2018 consists of the following 3 parts. It replaces the first edition (ISO 15835:2009) consisting only of 2 parts.

- ISO 15835-1:2018 Requirements
- ISO 15835-2:2018 Test methods
- ISO 15835-3:2018 Conformity assessment scheme.

For testing strength, ductility and slip under static loading, each test unit shall consist of couplers of the same splice type and size, and shall represent a maximum number of 10 000 couplers.

ISO 15835-3:2018 Conformity assessment scheme specifies rules for the certification and for the self-evaluation of couplers to be used for the mechanical splicing of steel reinforcing bars. It includes requirements for the control of the manufacturing process of the couplers and for the verification of their conformity in the form of mechanical splices.

Evaluation of the compliance of the products to the requirements of ISO 15835-3 may be done by one of the following methods.

- Product certification, described in Clause 5, is made by a certification body who inspects the manufacturing facility and collects test samples at regular periods;
- Verification of lots, described in Clause 6, is made by agreement between the supplier and the purchaser, and involves only testing of the exact lot that is going to be delivered to the purchaser.

Product certification has 3 stages – Qualification testing, Continuous third party surveillance and Factory production control. Sampling plan for each is given.

Self-declaration of product conformity through the testing of delivery lots as per ISO/IEC 17050. The manufacturer shall hold a valid certificate as per Clause 5 for the main range of the same type of couplers. The manufacturer can then use this method to certify some lots of products that are not entirely covered by the scope of the certificate. This method should, however, not be used for the majority of the production of the manufacturer. It is intended for minor deviations from the scope of the certification.

4 Design issues

When used in reinforcement, Grade B600 steel has the potential to impact design provisions and performance in reinforced concrete construction. Design provisions for computing flexural strength, axial load capacity, and shear strength may need to be adjusted for application to members reinforced with Grade B600 steel.

4.1 Flexural

The strength of reinforced concrete members under flexural, axial, or combined flexural and axial loading is a key consideration as provisions for these considerations would directly establish the size of the members. Understanding potential changes in the strength and behaviour of reinforced concrete members is necessary to ascertain that Grade B600 reinforcement is safe and economical to use in practice.

Flexural strength design using Grade B600 steel reinforcement is no different from that for normal strength steel reinforcement. The flexural resistance of the sections could be accurately computed using the well-established strain compatibility analysis method where the steel stress-strain relationship is idealized as being elastic-perfectly plastic and the ultimate concrete compressive strain equals to 0.0035 for normal strength concrete up to C50/60 and reduced to as low as 0.0026 for high strength concrete (C90/115). The basic flexural design equations for rectangular sections, in both cases of singly reinforced section and section with compression reinforcement, as given below will be used in some examples to illustrate the flexure design of both normal strength steel reinforcement and Grade B600 steel reinforcement.

<u>Singly reinforced rectangular section in bending at the ultimate limit state</u>

Design equations for bending

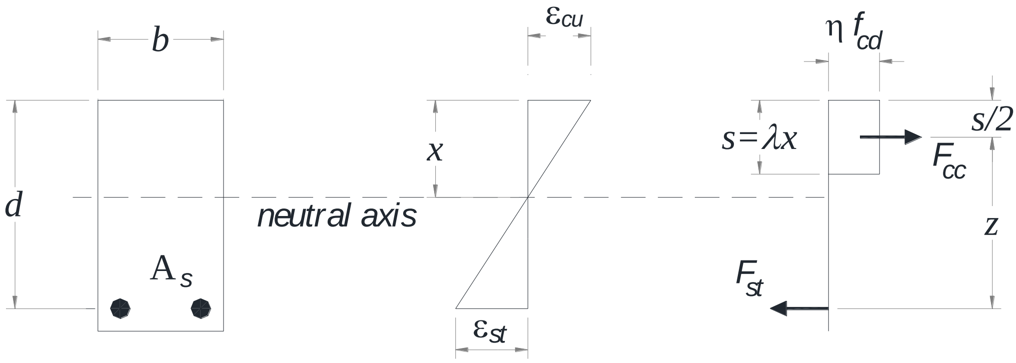


Figure 4-1: Stress and strain distribution in a singly reinforced rectangular section

As per definition, a section is balanced if the concrete strain reaches ε_{cu} simultaneously as the steel strain reaches design yield strain ε_{yd}, therefore the neutral axis depth for balance section at ultimate limit state is determined from the following equation:

$$\frac{x}{d} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}}$$

For normal strength concrete up to C50/60, ε_{cu} = 0.0035 thus for rebar grade B500 the design yield strain ε_{yd} = 0.00217 and x/d = 0.617, for rebar grade B600 the design yield strain ε_{yd} = 0.0026 and x/d = 0.573. However to ensure rotation at plastic hinge location EC2 (Clause 5.6.3) requires the maximum ratio x/d = 0.45 without moment redistribution for concrete class up to C50/60, i.e steel strain ε_s = 0.0043, and x/d = 0.35 for higher strength concrete. Correspondingly, the steel strain ε_s will be varying from 0.0058 for C55/67 to 0.0048 for C90/105. These steel strains are well above the nominal yield strain of Grade B500 (ε_y = 0.0025) and Grade B600 (ε_y = 0.003).

For equilibrium, the ultimate design moment shall be balanced by the moment of resistance of the section:

$$M = F_{cc}z = F_{st}z \qquad \textit{Equation 4-1}$$

where z is the level arm between the resultant forces F_{cc} and F_{st}

$$z = d\text{-}s/2 \qquad \textit{Equation 4-2}$$

The resultant force of concrete F_{cc} and depth of the rectangular stress block s are obtained from Figure 4-1 as follows:

$$F_{cc} = \eta f_{cd} bs = \eta \frac{\alpha_{cc} f_{ck}}{\gamma_c} bs \qquad \textit{Equation 4-3}$$

$$s = \lambda x \qquad \textit{Equation 4-4}$$

The resultant force of steel

$$F_{st} = A_s f_s / \gamma_s \qquad \textit{Equation 4-5}$$

According to Singapore National Annex (NA) to SS EN 1992-1-1, γ_c = 1.5, γ_s = 1.15 and α_{cc} can be taken as 0.85 for bending and compression. For concrete class up to C50/60 we have η = 1.0 and λ = 0.8, substitute into *Equation 4-3* with s obtained from *Equation 4-4,* we have

$$F_{cc} = 1.0 \times \frac{0.85 f_{ck}}{1.5} b \times 0.8x = 0.453 f_{ck} bx \qquad \textit{Equation 4-6}$$

and

$$M = F_{cc}z = 0.453 f_{ck} bx (d\text{-}0.4x) \qquad \textit{Equation 4-7}$$

Rearranging *Equation 4-7* and substituting $K = M/bd^2 f_{ck}$

$$0.181(x/d)^2 - 0.453 (x/d) + K = 0 \qquad \textit{Equation 4-8}$$

solving this quadratic equation:

$$x/d = 1.25 - \sqrt{1.5625 - 5.5147K} \qquad \textit{Equation 4-9}$$

As mentioned above, the maximum allowable ratio x/d without moment redistribution for normal strength concrete is 0.45 thus K_{bal} = 0.167

If $x/d \leq 0.45$ or $K \leq 0.167$ the rebar has yielded and the area of rebar can be computed from equation

$$M = F_{st}z = 0.87\, A_s f_{yk}\, (d\text{-}0.8x/2) \qquad \textit{Equation 4-10}$$

$$A_s = \frac{M}{0.87 f_{yk}\,(d - 0.4x)} \qquad \textit{Equation 4-11}$$

Replacing $M = Kbd^2 f_{ck}$ to *Equation 4-11* we have

$$A_s = \frac{Kbd^2 f_{ck}}{0.87 f_{yk}\,(d - 0.4x)} \qquad \textit{Equation 4-12}$$

$$\text{or} \quad \frac{A_s}{bd} = \frac{K}{0.87\,(1 - 0.4x/d)} \frac{f_{ck}}{f_{yk}} \qquad \textit{Equation 4-13}$$

From the condition x/d = 0.45 and K = 0.167 we also have maximum reinforcement ratio for singly reinforced section $A_s/bd = 0.234\, f_{ck} / f_{yk}$

For higher concrete class, the values of λ and η shall be computed from equations (3.20) and (3.22) of EC2 as follows:

$$\lambda = 0.8 - (f_{ck} - 50)/400, \ \eta = 1.0 - (f_{ck} - 50)/200 \qquad \textit{Equation 4-14}$$

from equilibrium of force we can obtain a general quadratic equation for x/d as follows

$$\frac{\alpha_{cc}\eta\lambda^2}{3}\left(\frac{x}{d}\right)^2 - \frac{\alpha_{cc}\eta\lambda}{1.5}\left(\frac{x}{d}\right) + K = 0 \qquad \textit{Equation 4-15}$$

At maximum ratio x/d = 0.35 we can obtain the corresponding factor K for different concrete classes with α_{cc} = 0.85 as shown in Table 4-1

Table 4-1 : Parameters λ, η, K for high strength concrete

f_{ck}	55	60	70	80	90
λ	0.7875	0.775	0.75	0.725	0.7
η	0.975	0.95	0.9	0.85	0.8
K	0.131	0.126	0.116	0.107	0.097

Replacing x/d = 0.35 and K factor from Table 4-1 into equation 3-13 we can also determine maximum reinforcement ratio for singly reinforced section A_s/bd for high strength concrete. Table 4-2 shows the maximum reinforcement ratio for singly reinforced section A_s/bd for each class of concrete with Grade B500 and Grade B600 reinforcement, assuming no moment redistribution.

Table 4-2 : Maximum reinforcement ratio for singly reinforced section without moment redistribution.

	Maximum reinforcement ratio for singly reinforced section $100A_s / bd$														
f_{ck} (MPa)	12	16	20	25	28	30	35	40	45	50	55	60	70	80	90
f_{yk} = 500 MPa	0.56	0.75	0.94	1.17	1.31	1.40	1.64	1.87	2.11	2.34	1.93	2.02	2.17	2.29	2.33
f_{yk} = 600 MPa	0.47	0.62	0.78	0.98	1.09	1.17	1.37	1.56	1.76	1.95	1.60	1.68	1.81	1.91	1.94

From the derivation of expressions for the design of singly reinforced concrete section, it appears that no modification of design equations is needed for use with Grade B600 reinforcement for strength design. However, the maximum reinforcement ratio that can be used for singly reinforced section without moment redistribution of Grade B600 reinforcement is smaller than that of Grade B500 reinforcement.

<u>Rectangular section with compression reinforcement at the ultimate limit state</u>

The design equations for rectangular section with compression reinforcement at the ultimate limit state for normal strength concrete can be found from Mosley *et. al* (2007) as follows:

$$A_s' = \frac{(K - K_{bal}) f_{ck} b d^2}{0.87 f_{yk}\, (d - d')} \qquad \textit{Equation 4-16}$$

$$A_s = \frac{K_{bal} f_{ck} b d^2}{0.87 f_{yk}\, z_{bal}} + A_s' \qquad \textit{Equation 4-17}$$

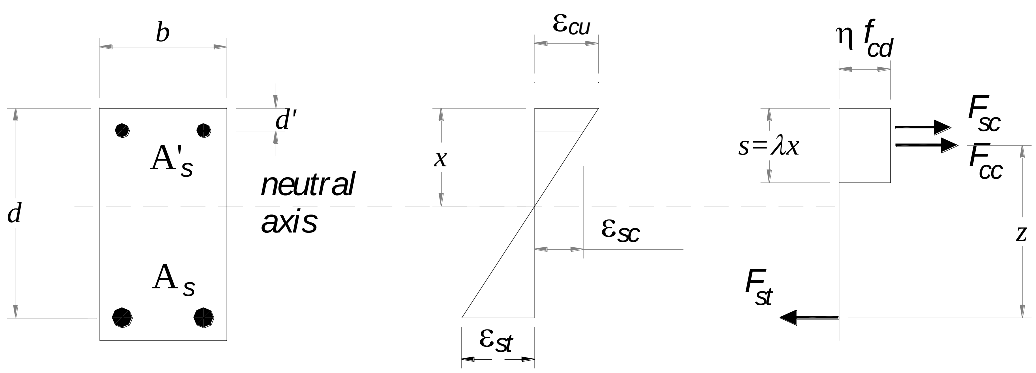


Figure 4-2 Stress and strain distribution in a doubly reinforced rectangular section

With the assumption that both compression and tension reinforcement have yielded, the condition for yielding of compression reinforcement is determined from the strain diagram in Figure 4-2 as follows:

$d'/x < 1 - \varepsilon_{sc} / \varepsilon_{cu}$ *Equation 4-18*

For normal strength concrete ε_{cu} = 0.0035, x/d = 0.45 and Grade B500 steel has design yield strain ε_{sc} = 0.00217 then

$d'/x < 1 - 0.00217/0.0035 = 0.38$, or $d'/d < 0.171$ *Equation 4-19*

For grade B600 steel $f_{yk} = 600\ N/mm2$, the design yield strain is ε_{yd} = 0.0026 therefore the condition for yielding of the compression steel, with normal strength concrete ε_{cu} = 0.0035, is

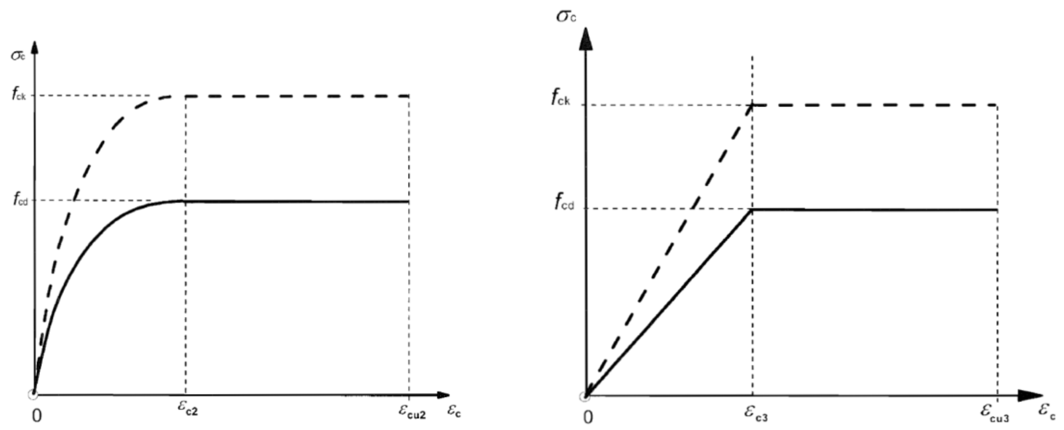
$d'/x < 1- 0.0026/0.0035 = 0.257$, or $d'/d < 0.116$ *Equation 4-20*

For high strength concrete, as ε_{cu} varies with concrete strength, $d'/x < 1 - \varepsilon_{sc} / \varepsilon_{cu}$ *Equation 4-18* should be used directly to check the condition for yielding of compression reinforcement.

Sample calculations on flexural design of beam are provided in examples 1.1 and 1.2

4.2 Compression and Bending

In EC2, the maximum strain of concrete in axial compression is 0.0035 for grade up to C50/60 and this reduces with higher strength concrete to 0.0026 for C90/105 (Table 3.1 of SS EN 1992), therefore when a normal strength rebar is used as the longitudinal reinforcement, the strain of the steel bar will be able to reach compression yield before the concrete reaches its maximum strain. But when Grade B600 rebar with higher yield strain is used, it may not yield even when the concrete reaches the maximum strain at the extreme fibre. The situation is even more limiting under pure or predominantly axial compression where the concrete strain at peak stress is 0.002 for C50/60 concrete, increasing to 0.0026 for C90/105 concrete. Hence, the Grade B600 steel bars may not reach yield potential before the concrete reaches the maximum stress under pure compression unless very high strength concrete is used.



(a) (b)

Figure 4-3 : EC2 stress-strain curves for concrete (a) Parabola-rectangle (b) Bi-linear stress-strain

In EC2, stress-strain relationships for the design of concrete are shown in Figure 4-3. The possible strain distributions in the ultimate limit state is shown in Figure 4-4. In EC2, for cross-sections that are subjected to bending and compression at the same time, the compressive strain in the concrete shall be limited to ε_{cu2} or ε_{cu3}. However, cross-sections or part thereof are subjected to approximately concentric loading, the compressive strain should be limited ε_{c2} or ε_{c3}.

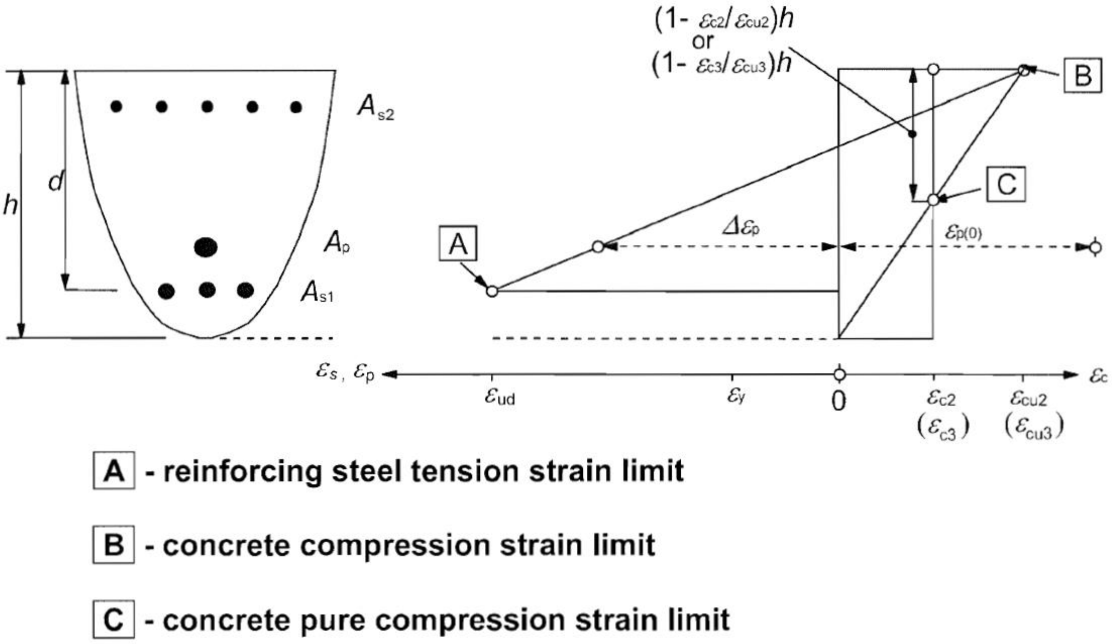


Figure 4-4: Possible strain distributions in the ultimate limit state (EC2 Figure 6.1)

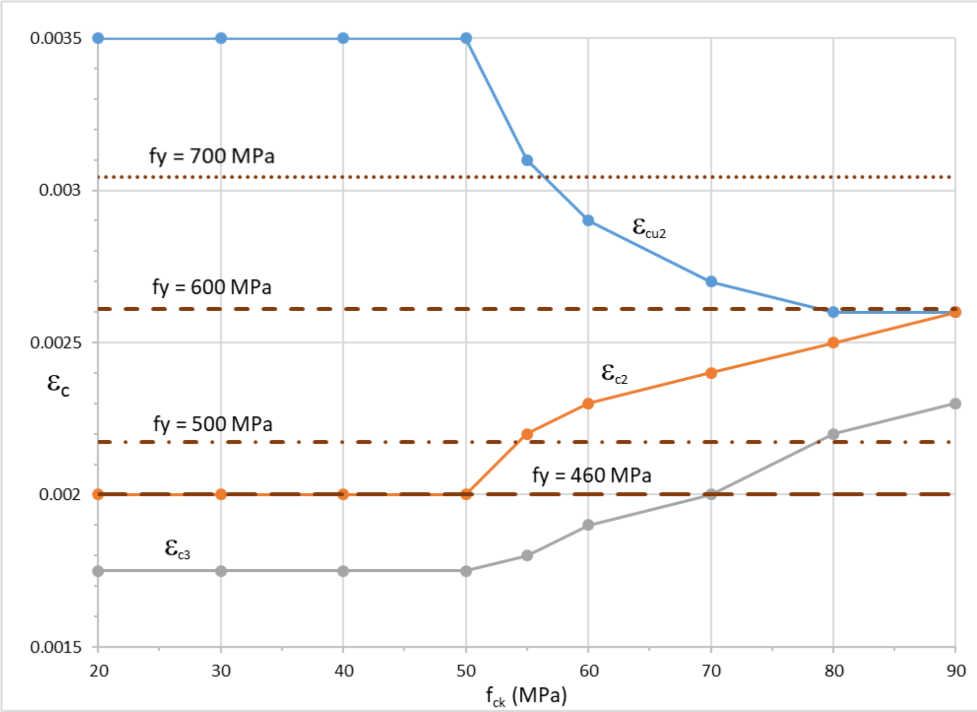


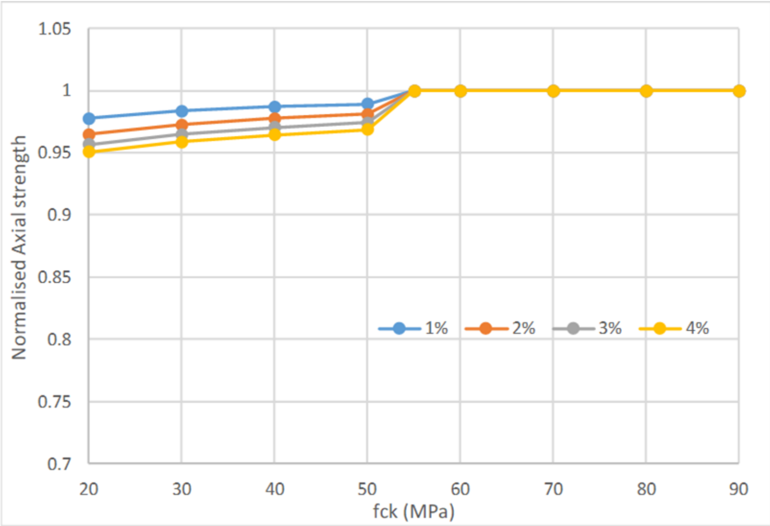
Figure 4-5: Concrete and rebar strains

In EC2, ε_{cu2} is 0.0035 for normal strength concrete (NSC) and reducing to 0.0026 for high strength concrete (HSC). As shown in Figure 4-5 and this compression strain of concrete for NSC is equal or higher than the design yield strain of steel of strength f_y = 700 MPa or lower and hence the steel has the potential to reach the yield strength under bending and compression load. Similarly, this is also true for f_y = 600 MPa for even HSC. As for pure compression load, if the strain of concrete ε_{c2} is equal or higher than the design yield strain of reinforcing bars for the case of steel with f_y = 460 MPa, the pure axial strength equation without influence of creep and drying shrinkage is

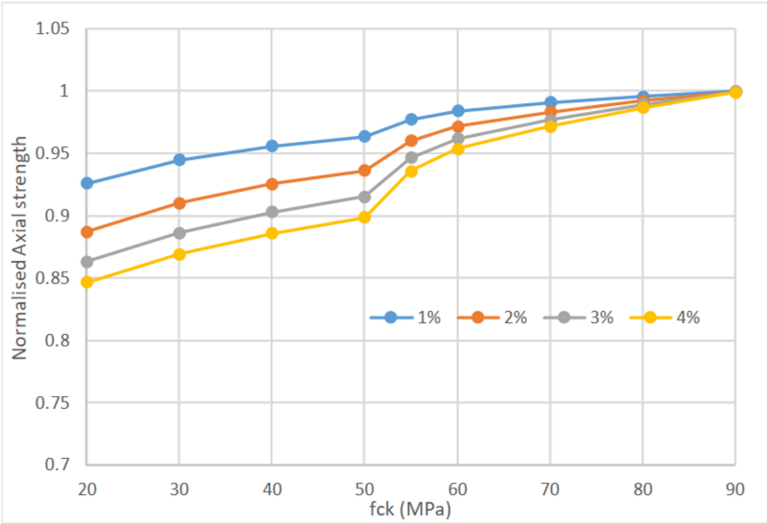
$$P_{uD} = 0.567\, f_{ck}\, (A_g - A_{st}) + 0.87\, f_{yk}\, A_{st} \qquad \text{Equation 4-21}$$

However, if the strain of concrete ε_{c2} is lower than the design yield strain of reinforcing bars, ε_{ykd}, for reinforcement equal or higher than 600 MPa yield strength, then it is not possible to realise all the potential strength of the steel reinforcement. This is also true for 500 MPa yield strength steel if concrete strength f_{ck} is less than 55 MPa. Therefore, when the influence of creep and drying shrinkage is neglected, the pure axial strength equation should be modified as following:

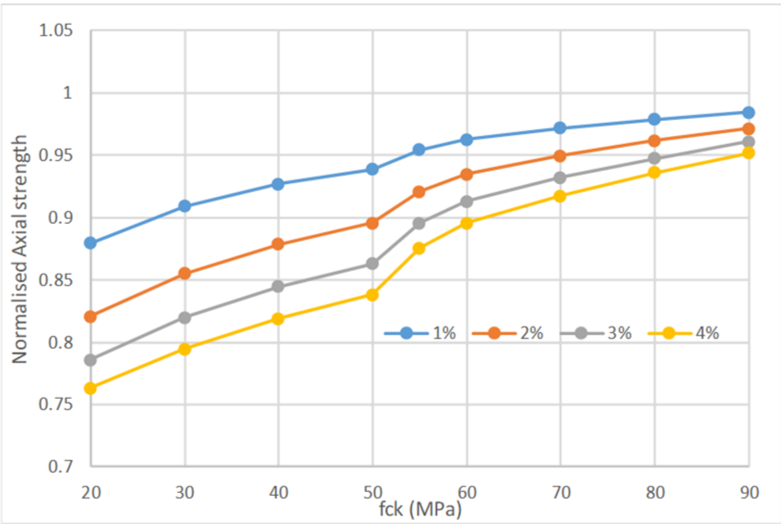
$$P_{uD} = 0.567\, f_{ck}\, (A_g - A_{st}) + E_s\, \varepsilon_{c2}\, A_{st} \quad \text{for } \varepsilon_{c2} < \varepsilon_{ykd} \qquad \text{Equation 4-22}$$



(a)



(b)



(c)

Figure 4-6 Normalized axial strength according to axial reinforcement ratio for (a) f_y = 500 MPa, (b) f_y = 600 MPa and (c) f_y = 700 MPa

Figure 4-6 shows the ratio of axial strength plotted against the concrete strength. The ratio of axial strength is calculated by the value of P_{uD} in Equation 4-22 divided by that in Equation 4-21. The analysis parameters were the compressive strength of concrete f_{ck} of 20 to 90 MPa, the yield strength of steel bars of 500 MPa, 600 MPa and 700 MPa, and the axial reinforcement ratio of 1% - 4%.

As shown in Figure 4-6, the axial strength ratio decreases as the axial reinforcement ratio increases, decreases as the yield strength of reinforcing bars increases, and increases as the concrete compressive strength increases.

In case of rebar with 500 MPa yield strength, when the compressive strength of concrete is less than 55 MPa, the normalised axial strength is below unity. This is because even though the value of ε_{c2} increases with concrete strength, for concrete strength lower than 55 MPa, the value of ε_{c2} is less than the design yield strength of Grade B500 MPa (ε_{c2} = 0.00217). Similarly, rebar with >600 MPa yield strength, the normalised axial strength for all concrete strength is below unity.

The above results are based on the analysis without considering the creep and the drying shrinkage in the reinforced concrete columns under the pure axial force. The effect of creep and shrinkage in the concrete will cause redistribution of forces between the rebar and concrete; the stress and strain in the rebar will be increased as a result. TTK (2017) proposed that a simple formula as shown in (Equation 4-23) can be used to represent the hypotheses:

$$\varepsilon_{c2,long} = (1 + \phi_{mcs})\,\varepsilon_{c,sust} + (\varepsilon_{c2} - \varepsilon_{c,sust}) = \phi_{mcs}\,\varepsilon_{c,sust} + \varepsilon_{c2} \qquad \textit{Equation 4-23}$$

$\varepsilon_{c2,long}$ is the concrete strain in considering creep and drying shrinkage; ϕ_{mcs} is the final increase coefficient in considering creep, drying shrinkage and rebar ratio; $\varepsilon_{c,sust}$ is the initial strain when the sustained load is applied.

If the fixed load was assumed to be 0.3 times of the factored load, $\varepsilon_{c,sust}$ could become 0.0006 or 0.3 times of the ε_{c2} which is 0.002 for C50 and below concrete.

The coefficient ϕ_{mcs} was taken as 2.0 and the additional strain would be 0.0012. The total strain of the concrete then became 0.0032 from the above equation. This value exceeds the design yield strain of 0.0026 of rebar with the yield strength of 600 MPa. Therefore, if the effects of creep and drying shrinkage are considered, the calculation formula of the pure axial strength in the Eurocode design could be applicable up to the Grade B600 rebar for all strength grades of concrete.

Similarly, SAH (2016) reported that the value of the additional strain can be assumed to be around 0.0005 and together with ε_{c2} of 0.0022 for C55/67 concrete gives a higher strain than design yield strain of 0.0026 for Grade B600 rebar.

The effects of creep and drying shrinkage would increase the concrete strain thereby enabling the steel to sustain a higher strain to reach yield. This phenomenon will be studied in more detail in the later section (see 4.9.1.1)

Under combined bending and compression, the maximum concrete compressive strain for normal strength concrete is higher than the strain at yield for Grade B600 steel reinforcement. Hence, there is no limit placed on the steel reinforcement to reach yield potential. The design of columns will not be different from those for the lower strength steel. Some design charts (Charts 2.1 to 2.8) for rectangular columns are created in the Examples section for concrete strengths of C28/35 and C90/105.

4.3 Shear

To avoid abrupt shear failure due to concrete crushing before the yielding of shear reinforcement and to control the diagonal crack width, it is recommended that the design strength of shear reinforcement of RC beams be limited. Based on limited available experimental study, the f_{ywk} be limited to 500 MPa for shear reinforcement. Similarly, owing to the lack of research data, f_{yt} should also be limited to 410 MPa for shear reinforcement designed for torsion.

4.4 Concrete strength

It is considered advantageous to use high-strength concrete in members that will use high-strength reinforcement. High concrete strength will reduce the required development and splice lengths of reinforcement, improve deformation capacity of flexural members, increase the shear strength of members, improve the strength of columns with high axial loads or combined axial load and flexure, increase the shear strength of joints in special moment frames, and reduce deflections (ATC-115(2014)).

4.5 Development Length

Lap splice issue

Developing the proper length of concrete-embedded rebar is crucial for obtaining its full tensile capacity. If the distance is less than the defined development length, the bar will pull out of the concrete. According to EC2, the development length concept is based on the attainable average bond stress over the length of embedment of the reinforcement. The development length is a function of steel bar yield stress, concrete compressive strength, and bar diameter. High strength concrete will reduce the required development and splice lengths due to the improved bond between concrete and rebar.

As the required anchorage and lap lengths are proportional with the stress in the reinforcement, higher yield strength reinforcement requires longer anchorage and lap lengths compared with lower yield strength reinforcement. In some cases, the required lap length of high strength rebar makes it unrealistic to use conventional lapping for connection of large diameter reinforcement.

Table 4-3 shows the Anchorage and lap lengths of Grade B600 reinforcement for different concrete classes, assuming the stress in the reinforcement reaches design stress of 522 N/mm^2, the concrete cover to all sides and distance between rebar are not less than 25mm and confinement effect is not considered ($\alpha_3 = \alpha_4 = \alpha_5 = 1.0$) . The anchorage and lap length of rebar in compression in this table shall be multiplied with a coefficient of 1.09 for rebar with 40mm diameter. The values in *Table* 4-3 are rounded up to the nearest 10mm.

Table 4-3 *:* Anchorage and lap lengths of Grade B600 reinforcement

		Bond condition	Reinforcement in tension, bar diameter, *f* (mm)								Rebar (compression)
			8	10	12	16	20	25	32	40	
Concrete class C20/25											
Anchorage length, l_{bd}	Straight bars only	Good	320	440	570	830	1090	1410	1800	2450	57ϕ
		Poor	450	630	810	1180	1550	2010	2570	3500	81ϕ
	Other bars	Good	310	570	680	900	1130	1410	1800	2450	57ϕ
		Poor	450	810	970	1290	1610	2010	2570	3500	81ϕ
Lap length, l_0	50% lapped in one location	Good	450	620	800	1170	1530	1990	2550	3460	80ϕ
		Poor	640	880	1150	1670	2190	2840	3640	4940	114ϕ
	100% lapped in one location	Good	480	660	850	1240	1630	2110	2700	3670	85ϕ
		Poor	680	940	1220	1770	2320	3020	3860	5240	121ϕ
Concrete class C25/30											
Anchorage length, l_{bd}	Straight bars only	Good	280	380	490	710	940	1220	1550	2110	49ϕ
		Poor	390	540	700	1020	1340	1730	2220	3010	70ϕ
	Other bars	Good	270	490	590	780	970	1220	1550	2110	49ϕ
		Poor	390	700	840	1110	1390	1730	2220	3010	70ϕ
Lap length, l_0	50% lapped in one location	Good	390	540	690	1010	1320	1720	2200	2980	69ϕ
		Poor	550	760	990	1440	1890	2450	3140	4260	98ϕ
	100% lapped in one location	Good	410	570	740	1070	1400	1820	2330	3160	73ϕ
		Poor	590	810	1050	1530	2000	2600	3330	4520	104ϕ

		Bond condition	Reinforcement in tension, bar diameter, ƒ (mm)								Rebar (compression)
			8	10	12	16	20	25	32	40	
Concrete class C28/35											
Anchorage length, l_{bd}	Straight bars only	Good	260	350	460	660	870	1130	1440	1960	45ϕ
		Poor	360	500	650	940	1240	1610	2060	2790	65ϕ
	Other bars	Good	250	450	540	720	900	1130	1440	1960	45ϕ
		Poor	360	650	770	1030	1290	1610	2060	2790	65ϕ
Lap length, l_0	50% lapped in one location	Good	360	500	640	940	1230	1590	2040	2770	64ϕ
		Poor	510	710	920	1330	1750	2270	2910	3950	91ϕ
	100% lapped in one location	Good	380	530	680	990	1300	1690	2160	2930	68ϕ
		Poor	540	750	970	1410	1860	2410	3080	4190	97ϕ
Concrete class C30/37											
Anchorage length, l_{bd}	Straight bars only	Good	250	340	440	630	830	1080	1380	1870	43ϕ
		Poor	350	480	620	900	1180	1540	1970	2670	62ϕ
	Other bars	Good	240	430	520	690	860	1080	1380	1870	43ϕ
		Poor	340	620	740	990	1230	1540	1970	2670	62ϕ
Lap length, l_0	50% lapped in one location	Good	340	480	610	890	1170	1520	1950	2640	61ϕ
		Poor	490	680	880	1270	1670	2170	2780	3770	87ϕ
	100% lapped in one location	Good	370	500	650	950	1240	1610	2060	2800	65ϕ
		Poor	520	720	930	1350	1770	2300	2950	4000	92ϕ
Concrete class C32/40											
Anchorage length, l_{bd}	Straight bars only	Good	240	320	420	610	800	1030	1320	1790	42ϕ
		Poor	330	460	590	860	1130	1470	1880	2560	59ϕ
	Other bars	Good	230	420	500	660	830	1030	1320	1790	42ϕ
		Poor	330	590	710	940	1180	1470	1880	2560	59ϕ
Lap length, l_0	50% lapped in one location	Good	330	460	590	860	1120	1460	1860	2530	59ϕ
		Poor	470	650	840	1220	1600	2080	2660	3610	83ϕ
	100% lapped in one location	Good	350	480	620	910	1190	1550	1980	2680	62ϕ
		Poor	500	690	890	1290	1700	2210	2820	3830	89ϕ
Concrete class C35/45											
Anchorage length, l_{bd}	Straight bars only	Good	220	300	390	570	750	970	1240	1690	39ϕ
		Poor	310	430	560	810	1070	1390	1770	2410	56ϕ
	Other bars	Good	220	390	470	620	780	970	1240	1690	39ϕ
		Poor	310	560	670	890	1110	1390	1770	2410	56ϕ
Lap length, l_0	50% lapped in one location	Good	310	430	560	810	1060	1370	1760	2380	55ϕ
		Poor	440	610	790	1150	1510	1960	2510	3400	79ϕ
		Good	330	450	590	860	1120	1460	1860	2530	59ϕ

		Bond condition	Reinforcement in tension, bar diameter, f (mm)								Rebar (compression)
			8	10	12	16	20	25	32	40	
	100% lapped in one location	Poor	470	650	840	1220	1600	2080	2660	3610	83ϕ
Concrete class C40/50											
Anchorage length, l_{bd}	Straight bars only	Good	200	280	360	520	690	890	1140	1540	36ϕ
		Poor	290	400	510	750	980	1270	1620	2200	51ϕ
	Other bars	Good	200	360	430	570	710	890	1140	1540	36ϕ
		Poor	280	510	610	810	1020	1270	1620	2200	51ϕ
Lap length, l_0	50% lapped in one location	Good	290	390	510	740	970	1260	1610	2180	51ϕ
		Poor	410	560	720	1050	1380	1790	2290	3110	72ϕ
	100% lapped in one location	Good	300	420	540	780	1030	1330	1700	2310	54ϕ
		Poor	430	590	770	1120	1470	1900	2430	3300	76ϕ
Concrete class C45/55											
Anchorage length, l_{bd}	Straight bars only	Good	190	260	330	480	640	820	1050	1430	33ϕ
		Poor	270	370	470	690	910	1170	1500	2040	47ϕ
	Other bars	Good	180	330	400	530	660	820	1050	1430	33ϕ
		Poor	260	470	570	750	940	1170	1500	2040	47ϕ
Lap length, l_0	50% lapped in one location	Good	260	360	470	680	900	1160	1490	2020	47ϕ
		Poor	380	520	670	970	1280	1660	2120	2880	67ϕ
	100% lapped in one location	Good	280	390	500	720	950	1230	1580	2140	50ϕ
		Poor	400	550	710	1030	1360	1760	2250	3050	71ϕ
Concrete class C50/60											
Anchorage length, l_{bd}	Straight bars only	Good	180	240	310	450	590	770	980	1330	31ϕ
		Poor	250	340	440	640	840	1090	1400	1900	44ϕ
	Other bars	Good	170	310	370	490	620	770	980	1330	31ϕ
		Poor	240	440	530	700	880	1090	1400	1900	44ϕ
Lap length, l_0	50% lapped in one location	Good	250	340	440	640	840	1080	1390	1880	44ϕ
		Poor	350	480	620	910	1190	1550	1980	2680	62ϕ
	100% lapped in one location	Good	260	360	460	680	890	1150	1470	1990	46ϕ
		Poor	370	510	660	960	1260	1640	2100	2850	66ϕ
Concrete class C55/67											
Anchorage length, l_{bd}	Straight bars only	Good	170	230	290	420	560	720	920	1250	29ϕ
		Poor	230	320	420	600	790	1030	1310	1780	41ϕ
	Other bars	Good	160	290	350	460	580	720	920	1250	29ϕ
		Poor	230	410	500	660	820	1030	1310	1780	41ϕ
		Good	230	320	410	600	780	1020	1300	1770	41ϕ

		Bond condition	Reinforcement in tension, bar diameter, *ƒ* (mm)								Rebar (compression)
			8	10	12	16	20	25	32	40	
Lap length, l_0	50% lapped in one location	Poor	330	450	590	850	1120	1450	1860	2520	58ϕ
	100% lapped in one location	Good	250	340	440	630	830	1080	1380	1870	43ϕ
		Poor	350	480	620	900	1190	1540	1970	2670	62ϕ
Concrete class C60/75 and above											
Anchorage length, l_{bd}	Straight bars only	Good	160	220	290	420	540	710	900	1220	29ϕ
		Poor	230	320	410	590	780	1010	1290	1750	41ϕ
	Other bars	Good	160	290	340	450	570	710	900	1220	29ϕ
		Poor	230	410	490	650	810	1010	1290	1750	41ϕ
Lap length, l_0	50% lapped in one location	Good	230	310	400	590	770	1000	1270	1730	40ϕ
		Poor	320	440	570	840	1100	1420	1820	2470	57ϕ
	100% lapped in one location	Good	240	330	430	620	810	1060	1350	1830	43ϕ
		Poor	340	470	610	890	1160	1510	1930	2620	61ϕ

<u>Compression and tension Splice Development (Mechanical Couplers)</u>

Grade B600 rebars require long anchorage and splice lengths. Based on EC2 formulae the anchorage and splice lengths of Grade B600 would be 20% longer than the Grade B500 rebars, which may be uneconomical or impractical, thus the designer may consider using mechanical splices and headed bars.

4.6 Serviceability

<u>Deflection and cracking</u>

As stated in the EC2 Commentary (2008) for a stress level of 200 N/mm^2 there is a probability of 95% that a maximum crack width smaller than 0.3 mm occurs, this also imply that the formulas for calculation of crack width shall aim at controlling the stress in tensile steel around 200 N/mm^2 if the allowable width of crack is 0.3mm, therefore there is no gain in performance under service condition when using high strength steel instead of normal strength steel. However any reduction in steel area would lead to higher steel stress under the same service condition, resulting in wider cracks than section with more steel area. The designer should make direct calculations to check the crack width under service condition.

To provide crack control at a reasonable bar spacing for members with increased cover, it will be necessary to limit the steel stress at service load to less than 310 MPa (up to 60% of f_y) recommended by EC2.

4.7 Reinforcement limits

Although no experimental validation is available, minimum reinforcement in accordance with EC2 provisions is deemed appropriate with Grade B600 reinforcement. To control cracking, as required by clause 7.3.1 (1) of the code, the minimum reinforcement area shall satisfy equation (7.1) as follows: $A_{s,min}\sigma_s = k_c k f_{ct,eff} A_{ct}$. In this equation, σ_s is the maximum stress permitted in the reinforcement immediately after formation of the crack. The code states that this value "may be taken as the yield strength of the reinforcement, f_{yk}. A lower value may be needed to satisfy the crack width limits according to the maximum bar size or spacing (see 7.3.3 (2))". When using Grade B600, if σ_s is taken as f_{yk} then the required value of $A_{s,min}$ become small. Hence, it is not recommended to use the yield strength f_{yk} for σ_s. It should be noted that in the calculation of crack width according to section 7.3.4 of the code, the crack width does not depend on yield strength of the reinforcement but only on the area and diameter of rebar.

To prevent a brittle failure and also to resist forces arising from restrained actions, the longitudinal reinforcement area of a beam or slab shall not be less than $A_{s,min} = 0.26\frac{f_{ctm}}{f_{yk}} b_t d$ but not less than $0.0013 b_t d$. The percentage of shear reinforcement in beam shall not be less than $\rho_{w,min} = (0.08\sqrt{f_{ck}})/f_{yk}$

For columns EC2 requires a minimum amount of longitudinal reinforcement of 0.2% A_g or 0.1 N_{ed}/f_{yd} whichever is greater, with A_g is gross cross-section area of column, N_{ed} is the axial force in the column.

4.8 Moment redistribution

4.8.1 General

Moment redistribution often provides for reserve capacity in members (or structures) in the event of overload. At present, no test data are available to judge if moment redistribution is applicable to members with Grade B600 reinforcements.

The neutral axis depth is considered the best parameter for quantifying the moment redistribution. EC2 states the permissible amount of redistribution depends on the tensile strain of the longitudinal reinforcement at the extreme layer, with the maximum amount being 30%, but it restricts the limitation of neutral axis depth to the effective depth x_u/d to a small value, intend to increase the maximum strain of tension reinforcement to be considered sufficient for rotational capacity.

4.8.2 Comparison of design parameters between EC2 recommended values and the NA to SS EN 1992-1-1

The EC2 allows the use of rebar yield strength of up to 600 MPa and a moment redistribution up to 30% for class B and C and up to 20% for class A reinforcement. In Clause 5.5, a different ratio of the redistributed moment to the elastic bending moment, δ, is stipulated for various steel Classes A, B and C. The ratio of neutral axis depth at ultimate limit state after redistribution, x_u , to the effective depth, *d,* is a direct factor to control the δ to ensure sufficient ductility for rotational capacity.

The Singapore NA to EC2 have allowed the use of rebar strength up to 600 MPa but it only specified the values of parameters for calculation of moment redistribution for steels with $f_{yk} \leq$ 500 MPa. For steel with f_{yk} > 500 MPa it is only stated that more restrictive values than those given for steels with $f_{yk} \leq$ 500 Mpa may need to be used.

Table 4-4 shows the comparison of moment redistribution and design parameters between EC2 recommended values and the NA to SS EN 1992-1-1 Classes B and C and concrete class up to C50/60.

Table 4-4 Moment redistribution and design parameters for concrete class up to C50/60 and reinforcement class B and C

Redistribution (%)	δ	x_u/d	ε_{su}	K_{bal}
Based on EC2 (k1 = 0.44, k2 = 1.25)				
0	1	0.448	0.0043	0.167
5	0.95	0.408	0.0051	0.155
10	0.9	0.368	0.0060	0.142
15	0.85	0.328	0.0072	0.129
20	0.8	0.288	0.0087	0.116
25	0.75	0.248	0.0106	0.101
30	0.7	0.208	0.0133	0.086
Based on NA to SS EN 1992-1-1 (k_1 = 0.4, k_2 = 1.0)				
0	1	0.45	0.0043	0.167

5	0.95	0.45	0.0043	0.167
10	0.9	0.45	0.0043	0.167
15	0.85	0.45	0.0043	0.167
20	0.8	0.4	0.0053	0.152
25	0.75	0.35	0.0065	0.136
30	0.7	0.3	0.0082	0.120

Note: For class A reinforcement, maximum percentage of redistribution is 20% only.

In Table 4-4 K_{bal} is the ratio defined as follow:

$$K_{bal} = M_{bal} / bd^2 f_{ck}$$

in which: M_{bal} is the maximum moment of resistant for a singly reinforced section

d is the effective depth of the section

b is the width of the section

f_{ck} is the characteristic cylinder strength of concrete

It can be seen from Table 4-4 that the EC2 recommendations require more ductile behaviour for moment redistribution than NA to SS EN 1992-1-1 does. For example at 15% moment redistribution, the reinforcement should reach a strain not less than 0.0072 compared with only 0.0043 in SS EN 1992-1-1. As the parameters k_1 and k_2 in SS EN 1992-1-1 only recommended for steel with $f_{yk} \leq 500$ MPa it is suggested that for steel with $f_{yk} > 500$ MPa the parameters recommended by EC2 should be used.

For higher strength concrete ($f_{ck} > 50$ MPa) the parameters in Table 4-4 will be changed according to the concrete strength as can be seen in *Table 4-5*. Figure 4-7 shows the variation of parameters x_u/d vs. ratio of the redistributed moment to the elastic bending moment δ for different concrete classes based on moment redistribution parameters recommended by EC2, while Figure 4-8 shows x_u/d vs. δ relationship followed Singapore Annex to EC2.

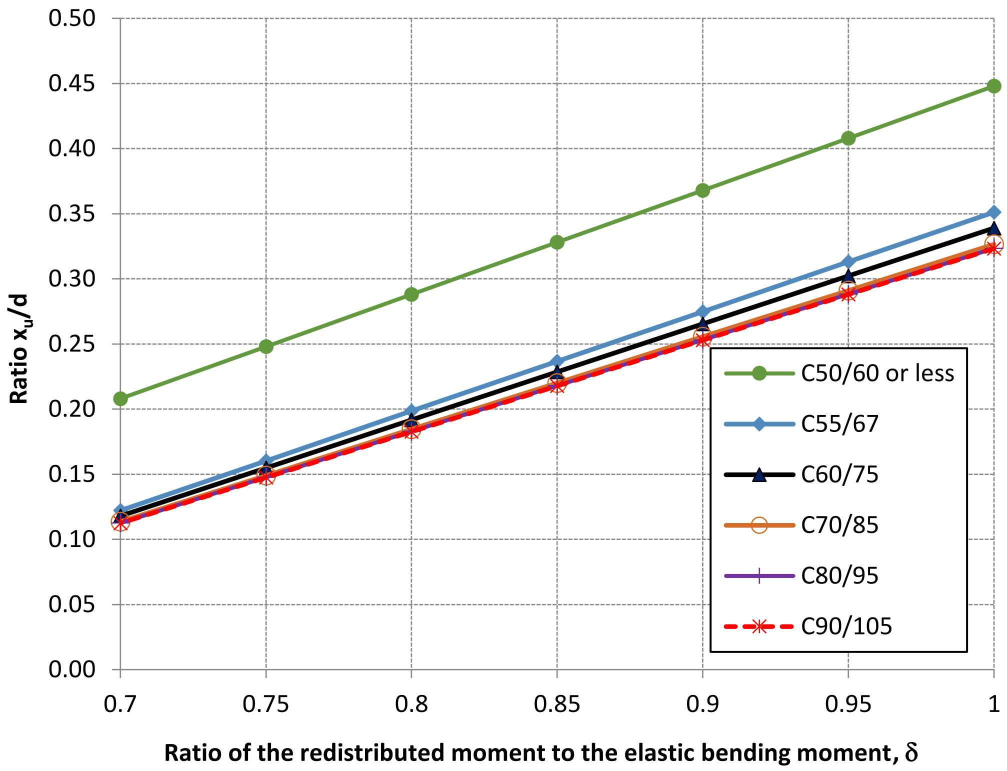


Figure 4-7 The ratio of x_u/d for moment redistribution design as recommended by EC2

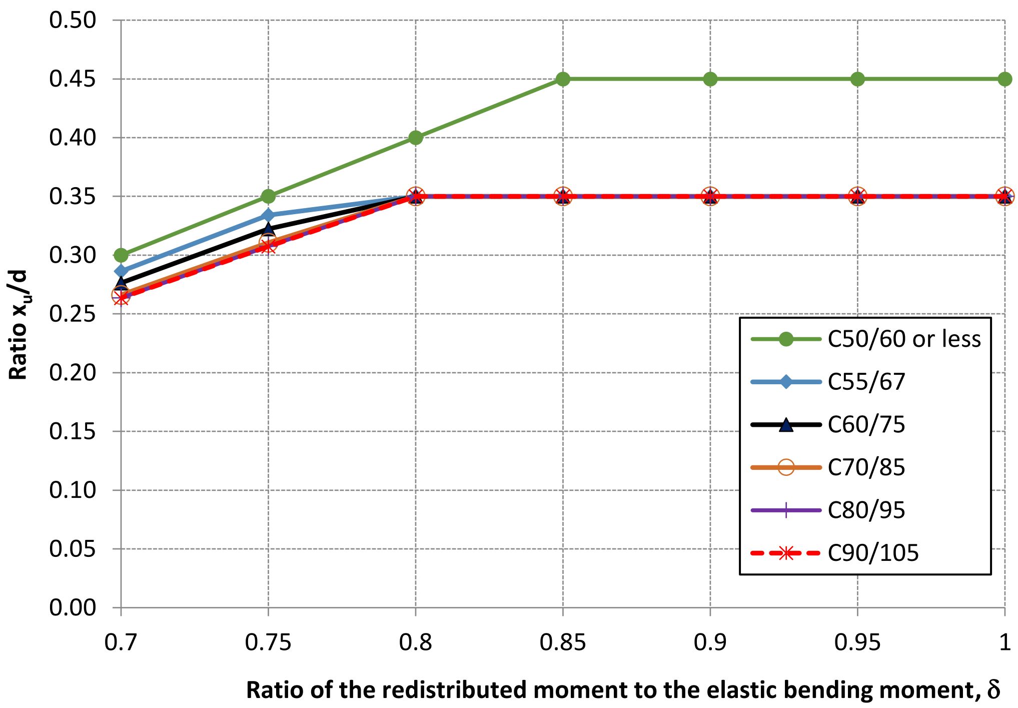


Figure 4-8 The ratio of x_u/d for moment redistribution design based on Singapore Annex to EC2

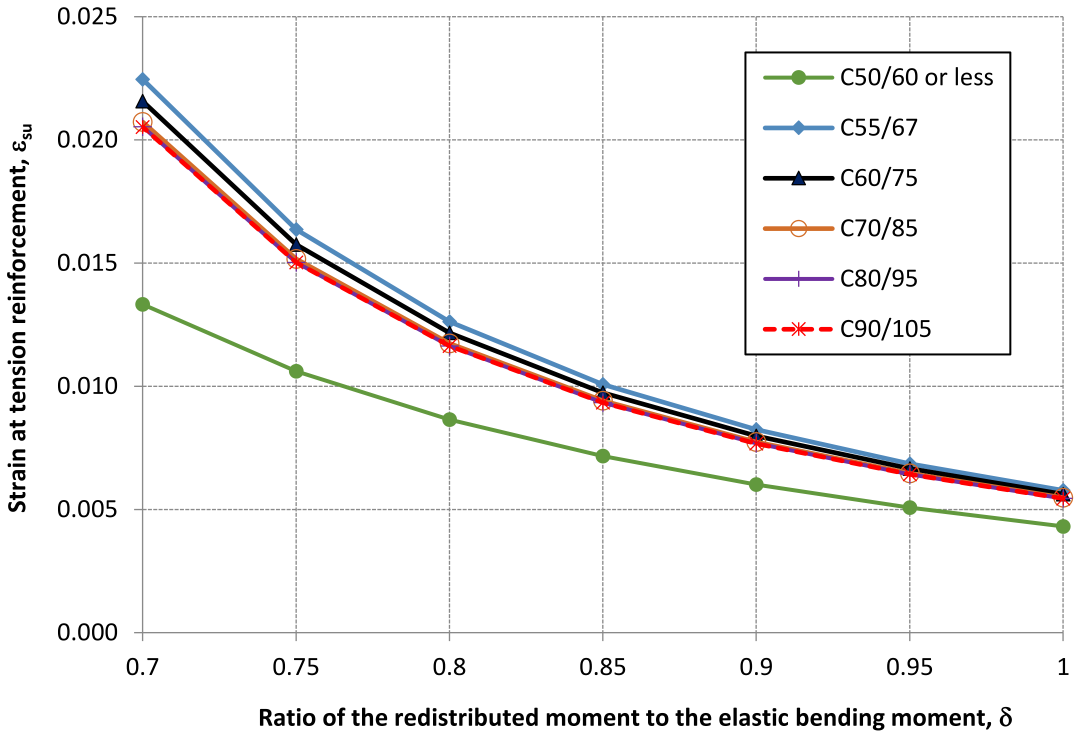


Figure 4-9 The strain at tension reinforcement ε_{su} vs δ as recommended by EC2

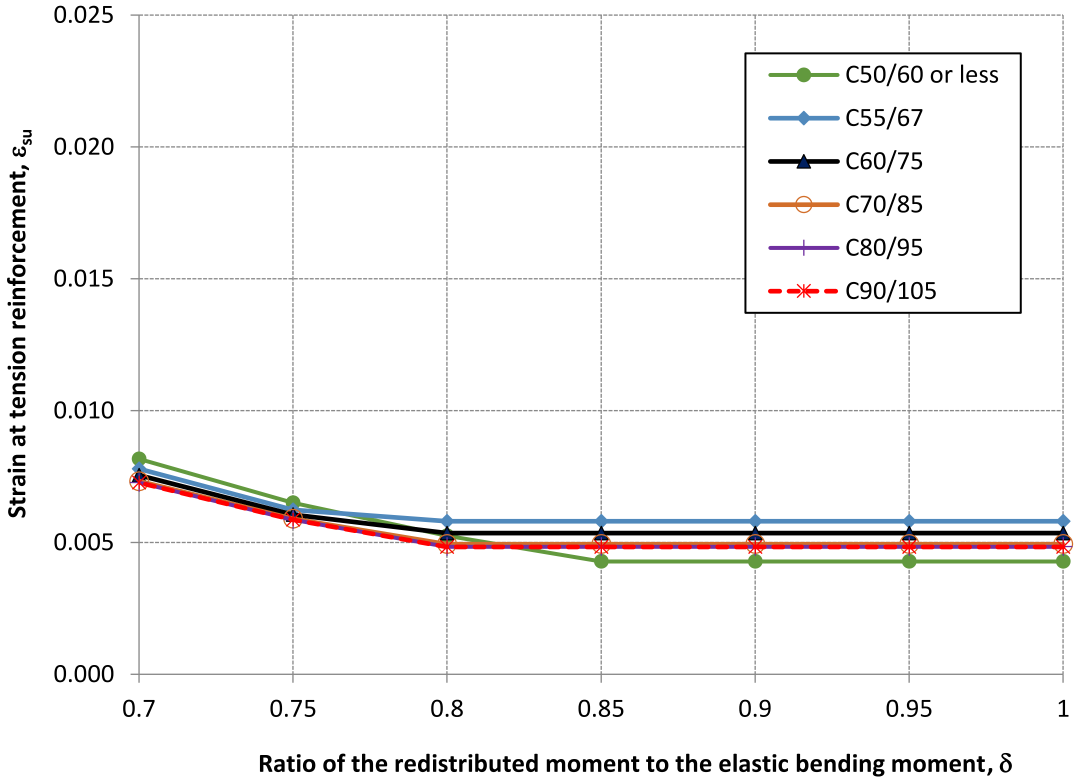


Figure 4-10 The strain at tension reinforcement ε_{su} vs δ relationship based on Singapore Annex to EC2

Figure 4-9 shows the strain at tension reinforcement ε_{su} vs. δ relationship based on moment redistribution parameters recommended by EC2, the required strain at tension reinforcement for high strength concrete (HSC) members with class higher than C50/60 is significantly higher compared with that for normal strength concrete members for the same ratio of the redistributed moment to the elastic bending moment. On the other hand, the strain at tension reinforcement ε_{su} computed based on Singapore Annex to EC2 as shown in Figure 4-10 is much smaller than that recommended by EC2. In other words, for HSC, the EC2 recommended parameters to design for moment redistribution would result in higher ductility or rotational capacity compared to those stipulated by UK and Singapore Annex.

It should be noted that for HSC the required strain of tension reinforcement at ultimate limit state exceeds 0.015 for 25% and 30% moment redistribution based on parameters recommended by EC2. Such a high value is not recommended in US practice (ACI ITG-6R-10), therefore it is recommended to limit the redistribution of moment in HSC to 20% only.

Table 4-5 Parameters for moment redistribution design for HSC

δ	f_{ck} = 55 MPa		f_{ck} = 60 MPa		f_{ck} = 70 MPa		f_{ck} = 80 MPa		f_{ck} = 90 MPa	
	x_u/d	K_{bal}	x_u/d	K_{bal}	x_u/d	K_{bal}	x_u/d	K_{bal}	x_u/d	K_{bal}
Based on EC2 recommended parameters (k_3 = 0.54, k_4= 1.25 (0.6+0.0014/ε_{cu}))										
1.00	0.350	0.131	0.339	0.123	0.326	0.114	0.323	0.107	0.323	0.102
0.95	0.313	0.119	0.302	0.111	0.291	0.103	0.288	0.097	0.288	0.092
0.90	0.275	0.107	0.265	0.099	0.256	0.092	0.253	0.086	0.253	0.082
0.85	0.237	0.093	0.228	0.087	0.220	0.080	0.218	0.075	0.218	0.071
0.80	0.198	0.080	0.192	0.074	0.185	0.068	0.183	0.064	0.183	0.060
0.75	0.160	0.065	0.155	0.061	0.149	0.056	0.148	0.052	0.148	0.049
0.70	0.122	0.051	0.118	0.047	0.114	0.043	0.113	0.040	0.112	0.038
Based on NA to SS EN 1992-1-1 (k_3 = 0.4, k_4 = 0.6+0.0014/ε_{cu})										
1.00	0.350	0.131	0.350	0.126	0.350	0.116	0.350	0.107	0.350	0.097
0.95	0.350	0.131	0.350	0.126	0.350	0.116	0.350	0.107	0.350	0.097
0.90	0.350	0.131	0.350	0.126	0.350	0.116	0.350	0.107	0.350	0.097
0.85	0.350	0.131	0.350	0.126	0.350	0.116	0.350	0.107	0.350	0.097
0.80	0.350	0.131	0.350	0.126	0.350	0.116	0.350	0.107	0.350	0.097
0.75	0.334	0.126	0.322	0.118	0.311	0.105	0.308	0.095	0.307	0.087

0.70	0.286	0.111	0.276	0.103	0.266	0.092	0.264	0.083	0.264	0.076

Comparing Singapore NA to SS EN 1992-1-1 and EC2 recommendations on moment redistribution, section designed according to EC2 recommendation will have more ductile or more rotational capacity. With shallower depth of neutral axis, it will ensure reinforcement will yield before concrete reaches ultimate strain 0.0035.

For steel stronger than B500, it is recommended that the parameters in Clause 5.5(4) on moment redistribution as suggested by the EC2 to be used instead of those given in the NA to SS EN 1992-1-1.

Sample calculations on moment redistribution for beams are provided in examples 3.1 and 3.2

4.9 Enhancement of concrete strain under pure compression

As explained in Section 4.2, when normal strength steel is used as the main reinforcement for column, the yield strain of the rebar is equal or lower than the strain of concrete under pure compression and both materials can reach the full potential strength. But, when the high strength steel with a higher yield strain is used, it may not yield even when the concrete reaches the maximum stress. In order to reach the yield potential of the steel reinforcement, the concrete compression strain has to be increased. The increase can be attained by either confinement or time-dependant effects.

4.9.1 By confinement

4.9.1.1 *Concrete Confined by Spiral Reinforcement*

Figure 4-11 shows the mechanism of passive confinement by reinforcements on RC circular columns and the formation of confining pressure due to hoop tension.

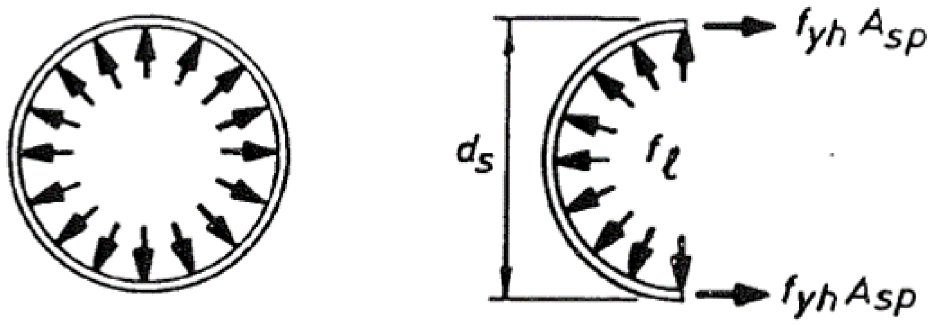


Figure 4-11: Effectiveness of Circular Confinement

Passive confinement relies on dilation of the core concrete as axial stress reaches and surpasses the unconfined concrete stress capacity. Confinement would be less effective for high-strength concrete than for normal-strength concrete. Concrete axial strength and strain capacity can be increased by applying compressive stresses in the directions transverse to the axial loading direction.

A more practical approach for application in concrete structures is to use transverse reinforcement to resist the dilation that occurs naturally when concrete is compressed.

Transverse reinforcement in a column serve three functions:

- Lateral support to the longitudinal reinforcement
- Confinement to the core concrete when the load supported by the column approaches it axial strength
- Shear reinforcement when the column is subjected to shear

Table 4-6 and *Table 4-7* summarise the equations proposed by different investigators to determine the peak strength and the corresponding strain enhancement due to the effects of circular confinement respectively.

Table 4-6: Strength Enhancement of Circular Confinement

Strength Enhancement of Circular Confinement	
$\frac{f'_{cc}}{f'_{co}} = 1 + K_1 \frac{f_t}{f'_{co}} = 1 + K_1 \frac{\rho_s f_{yh}}{2f'_{co}}$	Equation 4-24
Researcher	$\boldsymbol{K_1}$
Richard *et al*. (1929)	4.1
Balmer (1949)	5.6
Iyenger *et al*. (1970)	$4.6\left(1 - \frac{s}{d_s}\right)$ (MPa)
Fafitis and Shah (1985)	$1.15 + \frac{21}{f'_{co}}$ (MPa)
Watanabe *et al*. (1980)	$\frac{94}{\sqrt{f_{yh}}}\left(1 - \frac{s}{2d_s}\right)$ (MPa)
Sheikh and Uzumeri (1982)	$\frac{14.3}{\sqrt{\rho_s f_{yh}}}$

Mander *et al.* (1984)	$5.5K_e$

Table 4-7 **:** Strain Enhancement at Peak Stress due to Circular Confinement

Strain Enhancement at Peak Stress of Circular Confinement	
$\frac{\varepsilon_{cc}}{\varepsilon_{co}} = 1 + K_2 \frac{f_t}{f'_{co}} = 1 + K_2 \frac{\rho_s f_{yh}}{2f'_{co}}$	Equation 4-25
Researcher	$\boldsymbol{K_2}$
Iyenger *et al.* (1970)	$46\left(1 - \frac{s}{d_s}\right)$
Fafitis and Shah (1985)	15
Watanabe *et al.* (1980)	$\frac{914}{\sqrt{f_{yh}}}\left(1 - \frac{s}{2d_s}\right)$ (MPa)
Mander *et al.* (1984)	$27.5K_e$

4.9.1.2 *Concrete Confined by Rectangular Reinforcement*

The effectiveness of confinement in rectangular reinforcement is lesser than that for circular transverse steel, as explained in the theory section. As the hoops are pushed outward, the confinement on the sides of the rectangular hoops is limited and primarily only from bending stiffness; effective confinement only occurs at the corners of the hoops, as shown in *Figure 4-12*. Thus, passive confinement is mainly achieved by the diagonal expansion of core concrete. The following sections discuss the available stress-strain relationships for rectangular hoops with perimeter and internal ties.

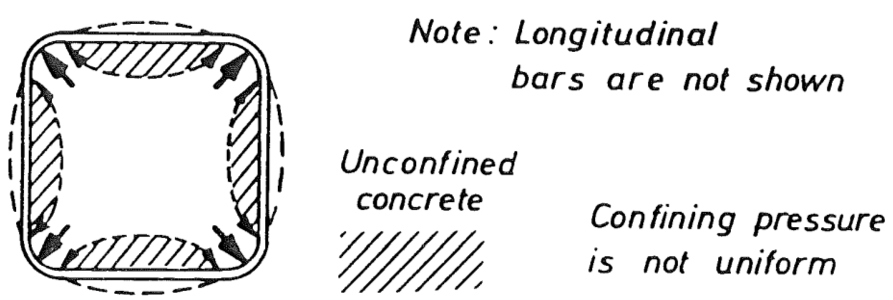


Figure 4-12: Effective Confinement of Rectangular Reinforcement

Cover concrete starts to spall at longitudinal strain around 0.004 evident by vertical splitting of the cover concrete and a reduction in load resistance. Failure occurs when the perimeter hoops fracture, accompanied by buckling of the longitudinal reinforcement and partial straightening of the 90 degree hooks on crossties of columns. The failure sequence is fairly typical for confined columns loaded monotonically in compression. Closely spaced transverse reinforcement acts to confine the core, imparting enhanced longitudinal strain capacity. It is established that rectangular reinforcement is less as effective as the circular spirals.

4.9.1.3 *Prediction of confinement of concrete using the Eurocode model*

According to EN 1992-1 the stress-strain curve of confined concrete can be represented by the same parabola-rectangular curve as the unconfined concrete but with increased characteristic strength of confined concrete as shown in equation (3.24) and (3.25) of the code and reproduced herewith.

$$f_{ck,c} = f_{ck}\,(1.000 + 5.0\ \sigma_2/f_{ck}) \text{ for } \sigma_2 \leq 0.05 f_{ck} \qquad \textit{Equation 4-26}$$

$$f_{ck,c} = f_{ck}\,(1.125 + 2.50\ \sigma_2/f_{ck}) \text{ for } \sigma_2 > 0.05 f_{ck} \qquad \textit{Equation 4-27}$$

and the strain at maximum stress $\varepsilon_{c2,c}$ and ultimate strain $\varepsilon_{cu2,c}$ are given in equation (3.25) and (3.26) of the code as

$$\varepsilon_{c2,c} = \varepsilon_{c2}\,(f_{ck,c}/f_{ck})^2 \qquad \textit{Equation 4-28}$$

$$\varepsilon_{cu2,c} = \varepsilon_{cu2} + 0.2\ \sigma_2/f_{ck} \qquad \textit{Equation 4-29}$$

where σ_2 is the effective lateral compressive stress at the ULS due to confinement.

ε_{c2} and ε_{cu2} are taken from table 3.1 of EN 1992-1 depending on concrete compressive strength, f_{ck}.

EN 1992-1 does not give any details how to determine the parameter σ_2 but it is given in EN 1998-3, applicable to rectangular section, as follows:

$$\sigma_2 = \alpha\ \rho_{sx} f_{yw} \qquad \textit{Equation 4-30}$$

where ρ_{sx} is the ratio of volume of transverse reinforcement to the volume of core concrete;

f_{yw} is the stirrup yield strength, and α is the confinement effectiveness factor

For rectangular section

$$\alpha = \left(1 - \frac{s}{2b_0}\right)\left(1 - \frac{s}{2h_0}\right)\left(1 - \frac{\sum b_i^2}{6h_0 b_0}\right)$$ *Equation 4-31*

where *s* is the spacing of transverse reinforcement,

b_0 and h_0 are the width and depth of the core concrete, measured to the centre line of perimeter link, respectively, and

b_i is the distance between consecutive engaged bars as illustrated in Figure 4-13.

Equation 4-30 can be extended to circular section with the confinement effectiveness factor determined from the following equation (EN 1998-1)

for circular hoops

$$\alpha = \left(1 - \frac{s}{2D_0}\right)^2$$ *Equation 4-32*

for circular spirals

$$\alpha = \left(1 - \frac{s}{2D_0}\right)$$ *Equation 4-33*

where D_0 is the diameter of the core concrete, measured to the centre line of the hoops or spirals.

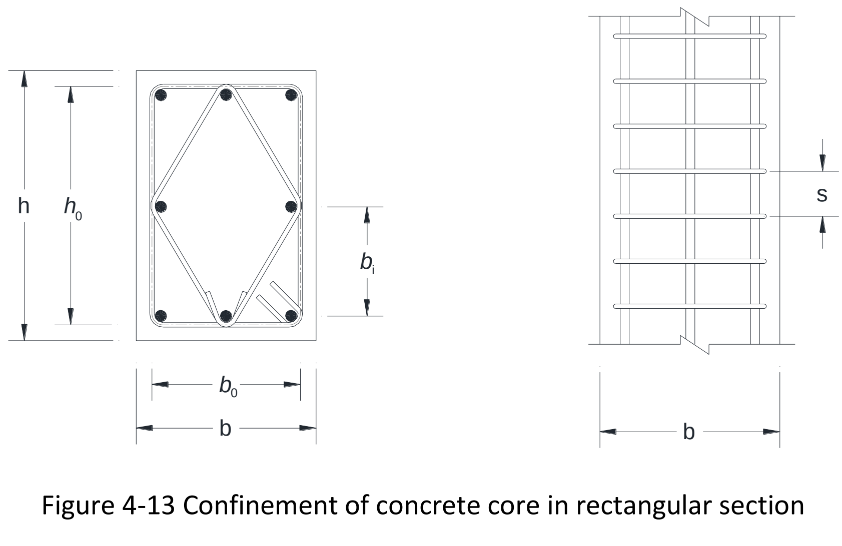


Figure 4-13 Confinement of concrete core in rectangular section

Sample calculations on confinement of concrete using the Eurocode model are provided in examples 4.1 to 4.4.

4.9.1.4 *Estimation of confining reinforcement amount to utilize the high strength reinforcement based on confinement mechanism given in EC2*

Considering a column subjected to concentric loading. Without considering the effect of creep and shrinkage the pure axial strength of the column is computed based on EC2 as rewritten below

$$P_{uD} = 0.567\, f_{ck}\, (A_g - A_{st}) + E_s\, \varepsilon_{c2}\, A_{st} \quad for\ \varepsilon_{c2} < \varepsilon_{ykd}$$

To fully utilize the high strength reinforcement, confinement from transverse reinforcement can be introduced so that the column can be loaded beyond the maximum unconfined strength so that the rebar can reach yield strength. However, beyond the unconfined strength, the concrete cover would be lost through spalling. Hence, the axial strength of the confined column in this stage is computed as

$$P_{uD,c} = 0.567(A_c - A_{st})f_{ck,c} + 0.87A_s f_{yk} \qquad \textit{Equation 4-34}$$

where

- $f_{ck,c}$ is the maximum confined concrete stress;

- A_c is the area of core concrete, measured to centre line of the perimeter hoops;

To maintain the axial strength of the column at least equal to the unconfined strength, we can determine the maximum confined concrete stress $f_{ck,c}$ from the condition $P_{0,c} = P_0$ as follows

$$0.567(A_g - A_{st})f_{ck} + A_{st}E_s\, \varepsilon_{c2} = 0.567(A_c - A_{st})f_{ck,c} + 0.87A_s f_{yk} \qquad \textit{Equation 4-35}$$

However the strain at maximum confined concrete stress $\varepsilon_{c2,c}$ also need to be not less than the design yield strain of the reinforcement ε_{ykd}. According to EN 1992-1-1 $\varepsilon_{c2,c} = \varepsilon_{c2}\, (f_{ck,c}/f_{ck})^2$ therefore this strain condition can be expressed as

$$\frac{\varepsilon_{c2}}{\varepsilon_{ykd}}\left(\frac{f_{ck,c}}{f_{ck}}\right)^2 = 1 \qquad \text{or } f_{ck,c} = f_{ck}\sqrt{\frac{\varepsilon_{ykd}}{\varepsilon_{c2}}} \qquad \textit{Equation 4-36}$$

The required maximum confined concrete stress $f_{ck,c}$ shall be the greater value obtained of those from Equation 2-27 and Equation 2-28. Once $f_{ck,c}$ is determined, the required confining stress σ_2 can be computed based on Equation 4-30 and the amount of confining reinforcement can be derived depending on configuration and yield strength of transverse reinforcement. The derivation of amount of confining reinforcement for some particular cases is as follows:

Case 1: For a square section confined with 1 Perimeter hoop + n Tie equally distributed in each direction, using the same hoop diameter:

$b_i = (b_0 - d_s - d_l)/(n+1)$ with d_l is the diameter of longitudinal rebar and d_s is the diameter of transverse rebar, for simplification on conservative side take $b_i = b_0/(n+1)$ thus

$$\alpha = \left(1 - \frac{s}{2b_0}\right)^2 \left(1 - \frac{2}{3(n+1)}\right) \qquad \textit{Equation 4-37}$$

$$\rho_{sx} = (4b_0 A_{sh} + 2nb_0 A_{sh})/sb_0^2 = [A_{sh}(4+2n)]/sb_0 \qquad \textit{Equation 4-38}$$

substitute ρ_{sx} and α into Equation 4-30 yields

$$\frac{\sigma_2}{f_{yw}} = \left(1 - \frac{s}{2b_0}\right)^2 \left(1 - \frac{2}{3(n+1)}\right)[A_{sh}(4+2n)]/sb_0 \qquad \textit{Equation 4-39}$$

$$\frac{\sigma_2 b_0}{\left(1 - \frac{2}{3(n+1)}\right)[A_{sh}(4+2n)]f_{yw}} s = 1 - \frac{s}{b_0} + \left(\frac{s}{2b_0}\right)^2 \qquad \textit{Equation 4-40}$$

$$1 - \left[\frac{1}{b_0} + \frac{\sigma_2 b_0}{\left(1 - \frac{2}{3(n+1)}\right)[A_{sh}(4+2n)]f_{yw}}\right]s + \frac{1}{4b_0^2}s^2 = 0 \qquad \textit{Equation 4-41}$$

Given b_0, A_{sh}, f_{yw}, σ_2 parameters we can solve this quadratic equation to give s value and then ρ_{sx} value.

Case 2: For a circular section confined with circular hoops, from *Equation 4-32*:

$$\alpha = \left(1 - \frac{s}{2D_0}\right)^2$$

$$\rho_{sx} = \frac{4\pi D_0 A_{sh}}{s\pi D_0^2} = \frac{4A_{sh}}{sD_0} \qquad \textit{Equation 4-42}$$

substitute ρ_{sx} and α into *Equation 4-30* yields

$$\frac{\sigma_2}{f_{yw}} = \left(1 - \frac{s}{2D_0}\right)^2 \frac{4A_{sh}}{sD_0} \qquad \textit{Equation 4-43}$$

$$\frac{\sigma_2 D_0}{4A_{sh}f_{yw}} s = 1 - \frac{s}{D_0} + \left(\frac{s}{2D_0}\right)^2 \qquad \textit{Equation 4-44}$$

$$1 - \left[\frac{1}{D_0} + \frac{\sigma_2 D_0}{4A_{sh}f_{yw}}\right]s + \frac{1}{4D_0^2}s^2 = 0 \qquad \textit{Equation 4-45}$$

Given D_0, A_{sh}, f_{yw}, σ_2 parameters we can solve this quadratic equation to give s value and then ρ_{sx} value.

Sample calculations on confining reinforcement amount to utilize the high strength reinforcement based on confinement mechanism given in EC2 are provided in examples 5.1 and 5.2

4.9.2 By time-dependant effects

4.9.2.1 *General*

A concrete element when kept under sustained load presents progressive strain over time, associated to the creep. In reinforced concrete columns, such deformations cause the stress increase in the steel bars of the reinforcement and may induce the material to undergo the yielding phenomenon. The time-dependent behavior of concrete due to its creep and shrinkage properties exerts a considerable influence on the performance of concrete structures, which may cause additional strain and stress redistribution (Manasseer and Lam, 2005).

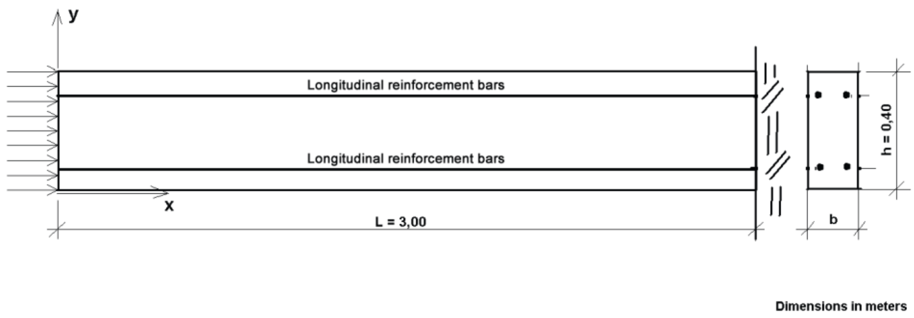
ACI 318-11 does not explicitly address the use of high strength reinforcing materials. The use of a higher compression yield stress can be justified by considering the long term redistribution of creep and shrinkage strains from the concrete to the reinforcement. Although ACI 318-11 does not address this phenomenon, Section 9.5.2.5 acknowledges it indirectly by allowing for a reduction in the long term component of deflections based upon the redistribution of creep and shrinkage strains to compression reinforcement

In reinforced concrete columns, creep and shrinkage lead to gradual load transfer from concrete to reinforcement. Assuming that cross sections remain flat caused by small strains due to creep and shrinkage under load, the stresses decrease in the concrete and increase in the reinforcing bars over time. A concrete element when kept under sustained load presents progressive strain over time, associated to the creep. In reinforced concrete columns, such deformations cause the stress increase in the steel bars of the reinforcement and may induce the material to undergo the yielding phenomenon (Rüsch *et al* 2011).

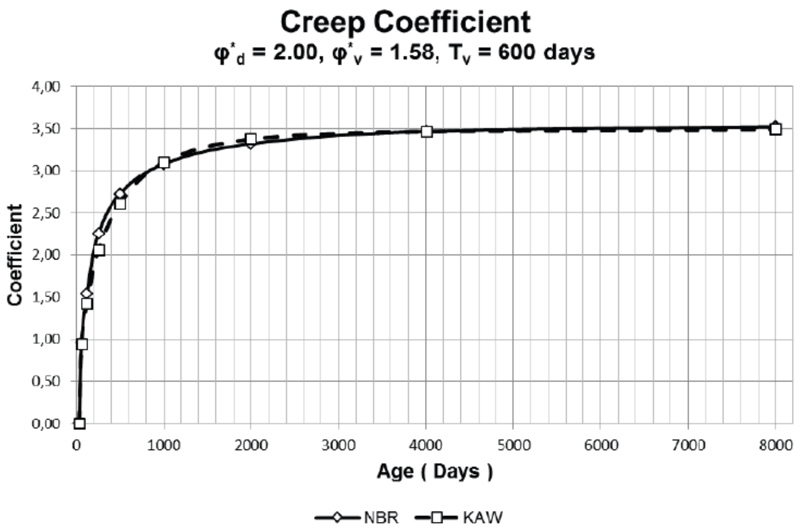
Based on experimental results of reinforced concrete columns, Takeuti (2003) reported that strain constraints introduced by reinforcing bars in columns should be considered, since they have a significant effect on rebar deformation.

Rossi and Maou (2013) conducted an experimental study of creep behaviour of concrete at variable stress levels (30, 50 and 70% of concrete compressive strength). They reported that a strain value of 0.005 is obtained without the failure of the concrete specimen after more than one year of loading at 70%, and that the strain is about 2.5 (250 %) times the strain at the peak of a classical compression test).

Madureira *et al* (2013) conducted a numerical study on creep strains on reinforced concrete columns. They presented a figure to show the curves of the creep coefficient evolution with time. Percentage change of total axial strain (with long-term effects of concrete) compared to conventional axial strain is about 200% in 400 days.



(a) Basic column model



(b) Creep coefficient over time

Figure 4-14: Numerical study on creep strains on reinforced concrete columns - Madureira et al (2013)

Ranaivomanana *et al* (2013) conducted an experimental study of creep behaviour of concrete at variable stress levels. They concluded that in terms of stress levels, non-linearity was found for compressive creep to arise somewhere between 30% and 50% of the high-strength concrete.

Zheng *et al* (2016) tested 12 groups of 100 x 100x 400 mm reinforced concrete specimens under variable compressive stress to investigate effects of parameters on the creep of RC columns and the sectional stress redistribution. They found that the ultimate creep value of axial compression RC columns decreases with the increase of longitudinal compression steel reinforcement ratio.

SAH (2016) reported on their study on the time-dependent concrete deformations and their impact, including the comparison between the methods from EC2 and ACI 318. The additional strain due to time effects were calculated for two cases: a simulation of a test-setup for a column subjected to concentric load until failure and a simulation of a load history for a multi-story building determining the additional strain in each level. For the first case of a concentric load until failure, the additional strain gained by the long term effects was approximately 0.1% after a year. For the second case, the strain development after 2 years for a 40 storey building including a construction time of one year was 0.05%.

Falkner et al (2008) has demonstrated that by taking creep and shrinkage of the concrete into consideration, the yield strength (up to including 670 MPa) of high strength steel reinforcement steel can be fully exploited.

TTK, (2017) in the study on the applicability of high strength rebar to concrete structures also proposed that the effect creep and shrinkage in increasing the compressive strain can be considered in column design. For example, if the permanent load is 30% of the factored load, the additional strain of 0.12% from the time effect can be attained. This enables the use of high strength steel of 600 MPa yield strength.

4.9.2.2 *Typical constituent of loads on columns*

The effect of creep in concrete is dependent on the level of sustained stress and the duration of the stress (the live to dead load ratios). In buildings, the sustained stress is due to the dead loads or permanent loads which comprises predominantly the self-weight of the structure and the finishes. During construction, the dead load on the columns are progressively added as the floors are constructed. The carrying capacity of the columns is to cater for the total load including the dead and imposed loads. For buildings, the provision for imposed loads is dependent of the intended usage level and to certain extend also affects the structural capacity of the building elements and hence the dead loads. A study was conducted to analyse the composition of loads in various buildings.

Table 4-8 Column loads for 21 buildings in Singapore

Type	Brief Description	$G_k/(1.35G_k+1.5Q_k)$			
		Min	Max	Average	Avg
Commercial/office	18 storey	0.47	0.61	0.52	0.52
Educational	Low-rise school	0.53	0.67	0.61	0.59
Educational	Low-rise school	0.53	0.68	0.59	
Educational	Low-rise school	0.56	0.66	0.61	
Educational	Low-rise school	0.57	0.69	0.61	
Educational	Low-rise school	0.56	0.70	0.62	
Educational	Low-rise school	0.51	0.68	0.60	
Educational	Low-rise school	0.53	0.68	0.60	
Educational	Low-rise school	0.51	0.59	0.56	
Educational	Low-rise school	0.46	0.66	0.59	
Educational	Low-rise school	0.53	0.63	0.57	
Educational	Low-rise school	0.52	0.65	0.59	
Educational	Low-rise school	0.48	0.68	0.57	
Hotel	10 storey	0.48	0.56	0.51	0.58
Hotel	6 storey	0.61	0.74	0.65	
Industrial	2 storey	0.16	0.49	0.36	0.42
Industrial	5 storey	0.47	0.57	0.52	
Industrial	13 storey	0.21	0.54	0.44	
Industrial	5 storey	0.30	0.46	0.34	
Residential	16 storey	0.44	0.66	0.59	
Residential	35 storey	0.55	0.70	0.62	

Table 4-8 shows the ratio of the unfactored design dead load against the total factored design load for 21 buildings in Singapore. There is a variation of values even within each category of building even though there is general trend. For example, the ratio tends to be lowest for industrial building because the imposed loads are much higher. On the other extreme, the ratio for the residential building is usually higher because the imposed loads are generally the lowest. Similarly, data from 14 buildings in Vietnam is shown in Table 4-9.

Table 4-9 Column loads for 21 buildings in Vietnam

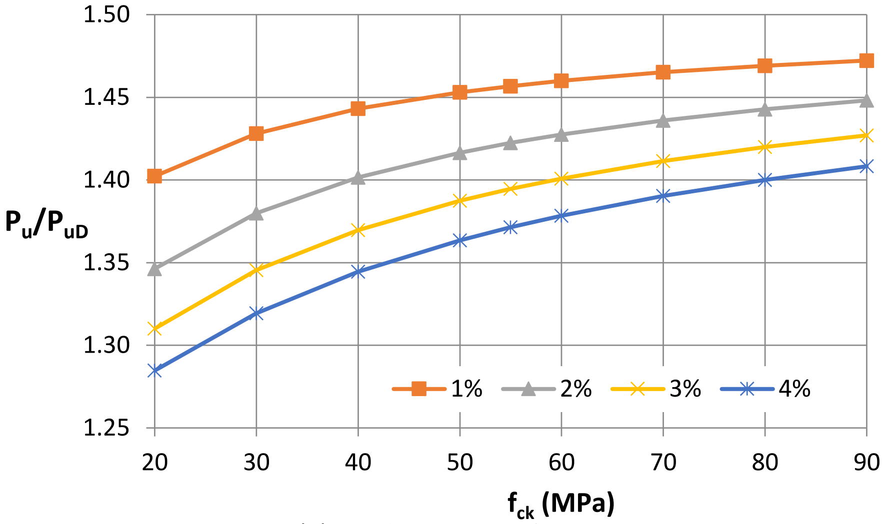
Type	Brief Description	$G_k/(1.35G_k+1.5Q_k)$
		(Average)
Residential	40 storey	0.47
Residential	40 storey	0.45
Residential	40 storey	0.45
Residential	41 storey	0.48
Residential	28 storey	0.46
Residential	33 storey	0.56
Residential	35 storey	0.56
Residential	37 storey	0.56
Residential	18 storey	0.46
Residential	38 storey	0.55
Residential	45 storey	0.51
Residential	25 storey	0.50
Commercial	28 storey	0.57
Average		0.51

It should be noted that the level of stress in column is not a function of that ratio because the denominator is the ultimate or total factored load. In design, the capacity or resistance of the structural element must be higher than the ultimate load (or Action). The level of stress should be based on the unfactored permanent load with respect to the capacity or resistance with material factor γ_m of unity.

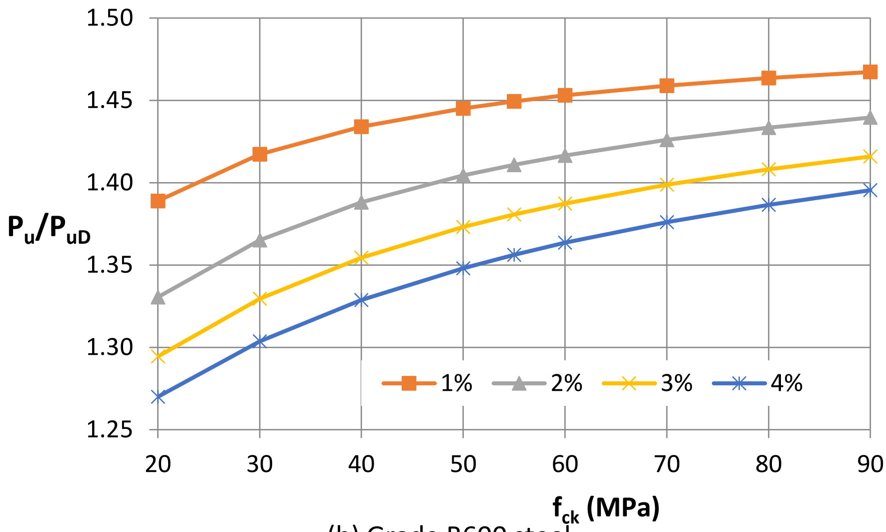
For reinforced columns under axial load only, the ultimate capacity P_u and design ultimate axial capacity P_{uD} can be expressed as

$$P_u = 0.85\, f_{ck}\, A_c + f_{yk}\, A_s \qquad \textit{Equation 4-46}$$

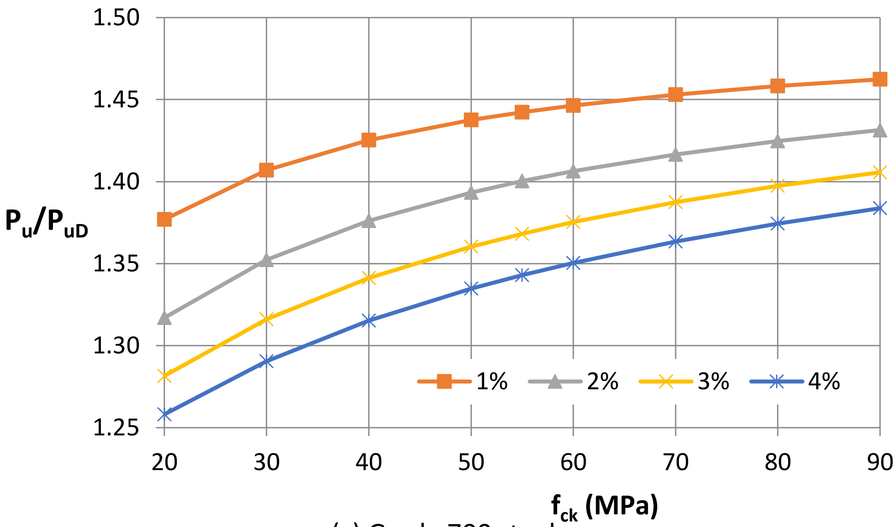
$$P_{uD} = 0.85\, f_{ck}\frac{A_c}{\gamma_c} + f_{yk}\frac{A_s}{\gamma_s} = 0.567\, f_{ck}A_c + 0.87 f_{yk}A_s \qquad \textit{Equation 4-47}$$



(a) Grade B500 steel



(b) Grade B600 steel



(c) Grade 700 steel

Figure 4-15 : Ratio of ultimate capacity against design ultimate capacity

Figure 4-15 shows the ratio of P_u against P_{uD} and this ratio is affected by different grades of steel, different concrete strength and different reinforcement ratios. To determine a conservative level of sustained stress in the column, the ratios in *Table 4-8* can be reduced by dividing using the highest ratio of 1.47 from Figure 4-15 which corresponds to f_{ck} = 90 MPa and 1% reinforcement.

4.9.2.3 *Effect of creep and shrinkage on load redistribution in columns*

As mentioned by Kong and Evans (1987), if concrete is subjected to a sustained stress, as usually the case in an actual RC column, then the total strain including elastic strain plus creep strain would increases with time, as illustrated in Figure 4-16, and there is a gradual redistribution of stress where the concrete sheds off the load it carries and this is picked up by the steel.

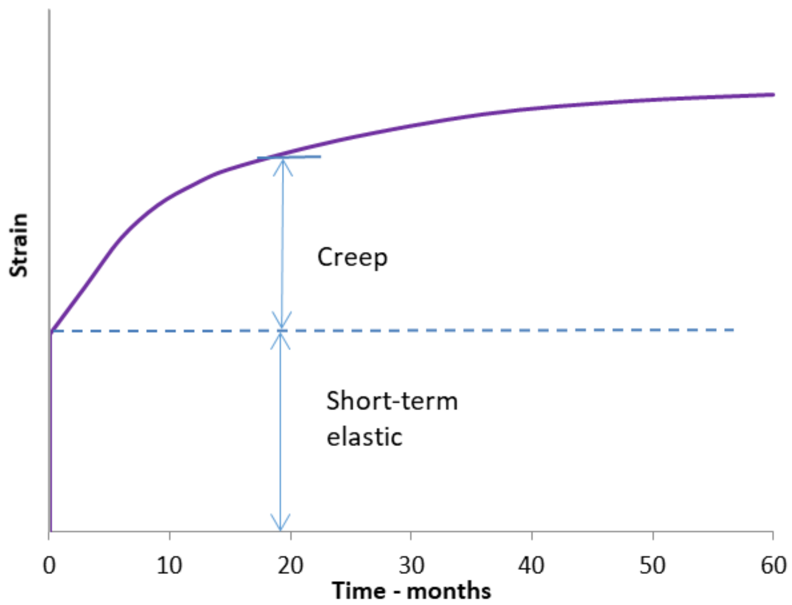


Figure 4-16 Typical increase of strain with time for concrete

According to SS EN 1992-1-1 (2010) the total creep strain $\varepsilon_{cc}(t,t_0)$ of concrete at time t for a constant compressive stress σ_c applied at the concrete age t_0 can be estimated from equation (3.6) of the code, by retaining variable t, as:

$$\varepsilon_{cc}(t,t_0) = \varphi(t,t_0).(\sigma_c/E_c) \qquad \textit{Equation 4-48}$$

where $\varphi(t,t_0)$ is the creep coefficient at time t and E_c is the tangent modulus of concrete, which may be taken as 1.05 E_{cm} , with E_{cm} is the secant modulus of concrete at concrete stress equal to $0.4f_{cm}$.

The creep coefficient depends on various factors and can be estimated in accordance with Appendix B of the code as follows:

$$\varphi\,(t,t_0) = \varphi_0.\beta_c(t,t_0) \qquad \textit{Equation 4-49}$$

Where φ_0 is the notional creep coefficient and may be estimated from

$$\varphi_0 = \varphi_{RH}.\beta(f_{cm}).\ \beta(t_0) \qquad \textit{Equation 4-50}$$

φ_{RH} is a factor to allow for the effect of relative humidity on the notional creep coefficient:

$$\varphi_{\mathrm{RH}} = \left[1 + \frac{1 + RH/100}{0.1\sqrt[3]{h_0}}\right] \qquad \text{for } f_{cm} \leq 35 \text{ MPa} \qquad \textit{Equation 4-51}$$

$$\varphi_{\mathrm{RH}} = \left[1 + \frac{1 + RH/100}{0.1\sqrt[3]{h_0}}\alpha_1\right]\alpha_2 \qquad \text{for } f_{cm} > 35 \text{ MPa} \qquad \textit{Equation 4-52}$$

RH is the relative humidity of the ambient environment in %.

$\beta(f_{cm})$ is a factor to allow for the effect of concrete strength on the notional creep coefficient:

$$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}} \qquad \textit{Equation 4-53}$$

$\beta(t_0)$ is a factor to allow for the effect of concrete age at loading on the notional creep coefficient:

$$\beta(t_0) = \frac{1}{0.1 + t_0^{0.20}} \qquad \textit{Equation 4-54}$$

h_0 is the notional size of the member in mm

$$h_0 = 2A_c/u \qquad \textit{Equation 4-55}$$

$\beta_c(t,t_0)$ is a coefficient to describe the development of creep with time after loading

$$\beta_c(t,t_0) = \left[\frac{t-t_0}{\beta_H+t-t_0}\right]^{0.3} \qquad \textit{Equation 4-56}$$

β_H is a coefficient depending on the relative humidity (*RH* in %) and the notional member size (h_0 in mm). It may be estimated from:

$$\beta_H = 1.5\ [1+(0.012\ RH)^{18}]h_0 + 250 \leq 1500 \qquad \text{for } f_{cm} \leq 35 \text{ MPa} \qquad \textit{Equation 4-57}$$

$$\beta_H = 1.5\ [1+(0.012\ RH)^{18}]h_0 + 250\ \alpha_3 \leq 1500\ \alpha_3 \qquad \text{for } f_{cm} > 35 \text{ MPa} \qquad \textit{Equation 4-58}$$

α_1, α_2, α_3 are coefficients to consider the influence of the concrete strength

$\alpha_1 = (35/f_{cm})^{0.7}$; $\alpha_2 = (35/f_{cm})^{0.2}$; $\alpha_3 = (35/f_{cm})^{0.5}$

Let's consider a case of RC column subjected to a concentric sustain load *P*. The concrete area is A_c and total longitudinal rebar area A_s. The modular ratio is defined as $\alpha_e = E_s / E_c$ then the concrete stress at the time of applying load t_0 shall be

$$\sigma_0 = \frac{N}{A_c + \alpha_e A_s} \qquad \textit{Equation 4-59}$$

and the stress in the reinforcing bar is

$$\sigma_{s0} = \alpha_e \sigma_0 \qquad \textit{Equation 4-60}$$

According to the item 3.1.4 (4) of code, if the concrete stress at age t_0 exceeds the value of 0.45 $f_{ck}(t_0)$ then creep non-linearity should be considered by using φ_{nl} (t,t_0) instead of φ (t,t_0)

$$\varphi_{nl}(t,t_0) = \varphi(t,t_0)\exp(1.5(k_\sigma - 0.45)) \qquad \textit{Equation 4-61}$$

where k_σ is the stress-strength ratio, taken as $\sigma_c / f_{ck}(t_0)$.

The short-term elastic strain of concrete and steel can be computed as

$$\varepsilon_0 = \sigma_0 / E_c \qquad \textit{Equation 4-62}$$

Due to creep, at time *t* the total concrete strain will reach $\varepsilon_c = \varepsilon_0 + \varepsilon_{cc}(t,t_0)$ and the concrete stress will be reduced to σ_c , which can be expressed using effective modular ratio α_e' as

$$\sigma_c = \frac{N}{A_c + \alpha_e' A_s} \qquad \textit{Equation 4-63}$$

The effective modular ratio α_e' shall be computed with the effective modulus of elasticity of concrete E_{ce} instead of original tangent modulus E_c

$$\alpha_e' = E_s / E_{ce}\,; \quad E_{ce} = \sigma_c / \varepsilon_c \qquad \textit{Equation 4-64}$$

An iterative procedure is needed to solve for the final creep strain and the redistributed stress in concrete σ_c and redistributed stress in reinforcing bar $\sigma_{s0} = \alpha_e' \sigma_c$ since the creep strain and the concrete stress are inter-dependent. This also reflects the redistribution of stress between concrete and rebar until it becomes stable. There is no influence of yield strength to the redistribution of stress except in cases the redistribution may lead to overstress of the rebar.

The shrinkage of concrete adds further stress redistribution between concrete and reinforcing bar. Shrinkage of concrete begins to take place as soon as the concrete is mixed and continue during the setting process. Even after the concrete has hardened, shrinkage continues as drying out persists over several months. According to SS EN 1992-1-1 the total shrinkage strain ε_{cs} is composed of two components, the drying shrinkage strain ε_{cd} and the autogenous shrinkage strain ε_{ca}.

$$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca} \qquad \textit{Equation 4-65}$$

The final value of the drying shrinkage strain, $\varepsilon_{cd,\infty}$ is equal to $k_h \cdot \varepsilon_{cd,0}$ where $\varepsilon_{cd,0}$ is the nominal unrestrained drying shrinkage value which may be taken from Table 3.2 and the coefficient k_h taken from Table 3.3 of the code.

The development of the drying shrinkage strain in time follows from equation (3.9) of the code:

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0} \qquad \textit{Equation 4-66}$$

$$\beta_{sd}(t, t_s) = \frac{(t-t_s)}{(t-t_s)+0.04\sqrt{h_0^3}} \qquad \textit{Equation 4-67}$$

where:

- t is the age of the concrete at the moment considered, in days;

- t_s is the age of the concrete (days) at the beginning of drying shrinkage (or swelling);

- h_0 is the notional size (mm) of the cross-section.

The autogenous shrinkage strain is given in equation (3.11) of the code as:

$$\varepsilon_{ca}(t) = \beta_{as}(t)\, \varepsilon_{ca}(\infty) \qquad \textit{Equation 4-68}$$

where:

$$\varepsilon_{ca}(\infty) = 2.5\,(f_{ck} - 10)\, 10^{-6} \qquad \textit{Equation 4-69}$$

and

$$\beta_{as}(t) = 1 - \exp(-0.2t^{0.5}) \qquad \textit{Equation 4-70}$$

where t is given in days.

An unrestrained concrete will have no stress but in a reinforced concrete column, the reinforcement bar resist the shrinkage and set up tensile stresses in the concrete and compressive stresses in the rebar itself. If the column is not restrained, as in the case it has just been cast without beam or slab connected at the top, then the rebar will subject to compressive strain ε_{sc} and the concrete will subject to tensile strain ε_{ct} as illustrated in *Figure* 4-17.

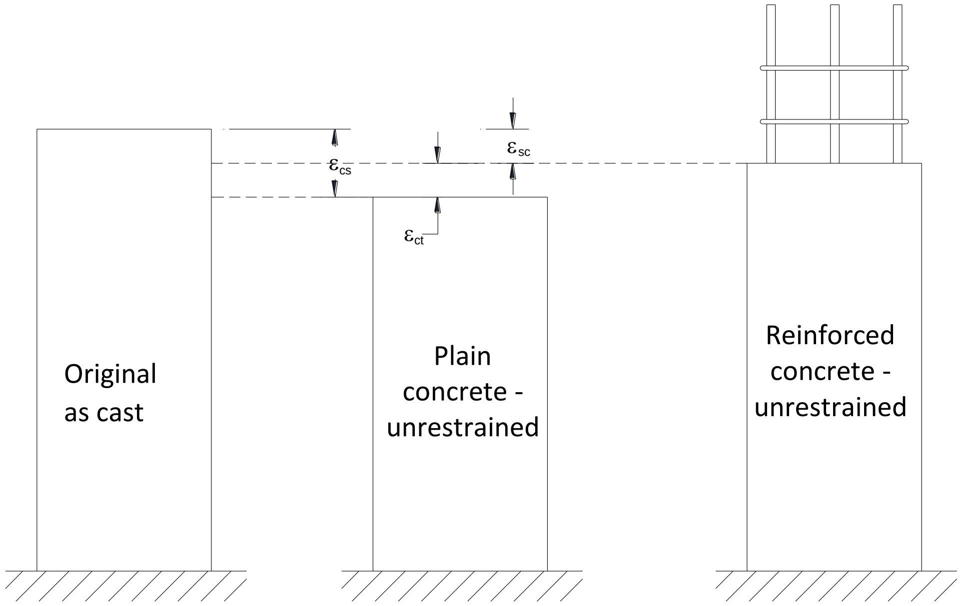


Figure 4-17 Effect of shrinkage strain

The basic equations for the case of unrestrained reinforced concrete subjected to shrinkage strain can be found from Mosley *et al* (2007) and reproduced as follow:

$$\varepsilon_{cs} = \varepsilon_{ct} + \varepsilon_{sc} = f_{ct} / E_{cm} + f_{sc}/E_s \qquad \textit{Equation 4-71}$$

where f_{ct} is the tensile stress in concrete area A_c
f_{sc} is the compressive stress in steel area A_s and
ε_{cs} is the total free shrinkage of concrete.

$$f_{ct} = \frac{A_s}{A_c} f_{sc} \qquad \textit{Equation 4-72}$$

$$f_{sc} = \frac{\varepsilon_{cs} E_s}{1 + \frac{\alpha_e A_s}{A_c}} \qquad \textit{Equation 4-73}$$

$$\varepsilon_{sc} = \frac{\varepsilon_{cs}}{1 + \frac{\alpha_e A_s}{A_c}} \qquad \textit{Equation 4-74}$$

α_e is the ratio E_s / E_{cm}

Sample calculations on effect of creep and shrinkage on load redistribution in columns are provided in Examples 6.1 and 6.2

5 Summary of design considerations

- Flexural strength design using Grade B600 steel reinforcement is no different from that for normal strength steel reinforcement. The flexural resistance of the section could be accurately computed using the well-established strain compatibility analysis method.
- Minimum reinforcement in accordance with EC2 provision is deemed appropriate with B600 reinforcement.
- Grade B600 steel bars may not reach compression yield potential before the concrete reaches the maximum stress under <u>pure compression</u> unless very high strength concrete is used.
- To avoid abrupt shear failure due to concrete crushing before the yielding of shear reinforcement and to control the diagonal crack width, it is recommended to have a limitation on the yield strength of shear reinforcement of RC beams. Based on available experimental study, the f_{ywk} is to be limited to 500 MPa for Grade B600 as shear reinforcement.
- Slabs and beams using Grade B600 rebars may result in lightly reinforced members with increased steel service stress. As there is no gain in performance under service condition when using high strength steel instead of normal strength steel, the designer should make calculations to check for crack width and deflection limits in serviceability limit states.
- Grade B600 steel bars require long anchorage and splice lengths thus the designer may consider using mechanical splices and headed bars.
- Currently, there is no product specifications for the rebar couplers to be stipulated for use. It is recommended that ISO 15835 be adopted for that purpose. For consistency in quality conformity as that recommended for the rebars in SS 560, the Conformity assessment scheme in ISO 15835 provides the rules for the certification and for the self-evaluation of couplers to be used for the mechanical splicing of steel reinforcing bars. It also includes requirements for the control of the manufacturing process of the couplers and for the verification of their conformity.
- On moment redistribution in beams, for steel stronger than Grade B500, it is recommended that the parameters in Clause 5.5(4) on moment redistribution as suggested by the EC2 to be used instead of those given in the NA to SS EN 1992-1-1.
- The use of confinement can enhance the ductility of concrete. It also increases the strain at peak concrete stress thereby matching or even exceeding that of the strain at yield of the steel reinforcement. However, the increase in strength of the confined concrete axial

capacity must be more than the loss of the strength of the unconfined concrete (especially the cover concrete).

- While the mechanism of confinement can be utilised to increase the strain at peak stress of concrete in compression, it entails increasing the amount of transverse reinforcement in the entire length of column. This increase in transverse steel quantity also reduces the construction productivity which together may offset any advantage of using high strength steel for the main reinforcement.
- The effects of time-dependant deformation such creep and drying shrinkage can cause redistribution of forces in RC columns and increases the strain of the concrete and steel. This enables the steel to sustain a higher strain to reach yield.

References

1. ACI 318 (2011). Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary, American Concrete Institute, Farmington Hills, MI.

2. ACI ITG-6R-10 (2010) Design Guide for the Use of ASTM A1035/A1035M Grade 100 Steel Bars for Structural Concrete, American Concrete Institute, Farmington Hills, USA, 94p

3. ATC-115 (2014) Roadmap for the use of high-strength reinforcement in reinforced concrete design, Appplied Technology Council

4. Al-Manaseer, A and J.-P. Lam, J-P (2005)" Statistical Evaluation of Shrinkage and Creep Models" ACI Materials Journal 102(3):170-176

5. Madureira E. L., Siqueira T. M., Rodrigues E. C. (2013) "Creep Strains on Reinforced Concrete Columns" IBRACON Structure and Materials Journal, 6(4), 537-560.

6. Falkner et al (2008), "The new reinforcement system; compression members with SAS 670 high strength reinforcement steel – Part 1 Development, testing, design, construction", Beton- und Stahlbentonbau 05/2008 No 614

7. Kong, F.K and Evans R.H (1987), "Reinforced and Prestressed Concrete", 3rd Edition, Chapman & Hall, 506pp.

8. Mander J.B. et al (1988), "Theoretical Stress ‐ Strain Model for Confined Concrete", Journal of Structural Engineering, Volume 114 Issue 8 - September 1988

9. Mosley, B., Bungey, J., and Hulse, R. (2007), "Reinforced Concrete Design to EC2", Six Edition, Palgrave MacMillan, 408 pp

10. Ranaivomanana, M, Multon, S and Turatsinze A (2013) "Tensile, Compressive and Flexural Basic Creep Of Concrete at Different Stress Levels" Cement and Concrete Research 52:1–10.

11. Rossi, P, Taihan, J.L., Maou F. L (2013) "Creep strain versus residual strain of a concrete loaded under various levels of compressive stress" Cement and Concrete Research, 51, 32-37.

12. Rüsch, H; Jungwirth, D; Hilsdorf, H K (2011) "Creep and Shrinkage: Their Effect on the Behavior of Concrete Structures" Published by Springer-Verlag New York Inc.

13. SAH (2016), "Time-Dependent concrete deformations and their impact - Comparison EC2 and ACI 318)", Stahlwerk Annahutte

14. TTK, (2017) "Study on Applicability of High Strength Rebar for Concrete Structures in Korea" Tokyo Tekko Co., Ltd. Mar 2017

15. Zheng W, Tang C and Liu Y (2016) "Creep Behavior of Reinforced Concrete Column in Consideration of Sectional Stress Redistribution" Journal of Building Structures. 37(5) 264-272.

Example calculations

1. Flexural Design of beam

Example 1.1 : Design of a singly reinforced rectangular section

The ultimate design moment to be resisted by the section in Figure 1 is 185 kNm. Determine the area of tension reinforcement (A_s) required given the characteristic material strengths are f_{yk} = 600 N/mm^2 for the reinforcement and f_{ck} = 25 N/mm^2 for the concrete.

Note: This example is similar to Example 4.1 in the text book by Mosley et al "Reinforced Concrete Design to EC2" but using high strength steel instead of f_{yk} = 500 N/mm^2 in that example.

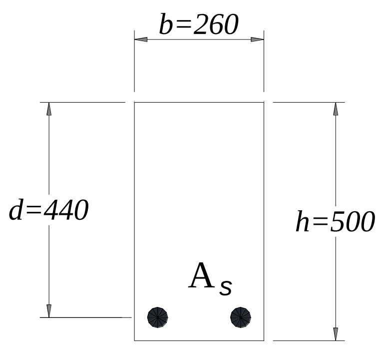


Figure 1. Section in example 1.1

Calculate $K = M/bd^2f_{ck} = 185 \times 10^6 / (260 \times 440^2 \times 25) = 0.147 < 0.167$

Therefore, compression steel is not required. From *Equation 4-9*

$$x/d = 1.25 - \sqrt{1.5625 - 5.5147K} = 0.383$$

$$x = 0.383 \times 440 = 168 \text{ mm}$$

The required steel area is

$$A_s = M / [0.87f_y (d\text{-}0.4x)] = 185 \times 10^6 / [0.87 \times 600 (440\text{-}0.4 \times 168)] = 951 \text{ mm}^2$$

Comment: Example 4.1 in the text book that uses f_{yk} = 500 N/mm^2 requires A_s = 1140 mm^2. Therefore, a saving of 17% in total steel area is achieved for this singly reinforced section when changing from grade B500 to grade B600 steel if only strength design is performed.

<u>Check crack width for the above example</u>

Assuming the moment under the action of the quasi-permanent load combination is M_{qp} = 0.7M =129.5 kNm, exposure class XC1 with maximum allowable crack width w_{max} = 0.3 mm according to NA to SS EN 1992-1-1, the creep coefficient φ = 2.85 for dry atmosphere condition (50%RH), and age of loading is 28 days. Using high bond bars for reinforcement.

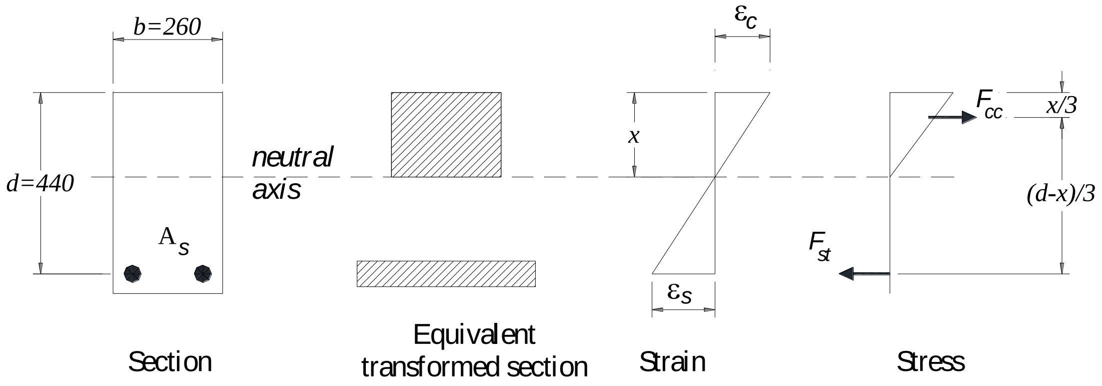


Figure 2 The stress-strain distribution in a cracked section for serviceability design checking

It should be noted that Singapore NA adopts the recommended values in EC2 for parameters k_3 and k_4 in calculation of crack width.

Case 1: using f_{yk} = 500 N/mm^2 , as mentioned from strength calculation the required rebar area is A_s = 1140 mm^2, choose 2ϕ25 + 1ϕ20 ($A_{s,prov}$ = 1296 mm^2) with bar spacing at 70mm.

From Table 3.1 of EC2, for f_{ck} = 25 N/mm^2 we have the following design parameters:

Mean compressive strength f_{cm} = 25+8 = 33 N/mm^2

Mean tensile strength $f_{ctm} = 0.3\, f_{ck}^{(2/3)}$ = 2.56 N/mm^2

Modulus of elasticity $E_{cm} = 22(f_{cm}/10)^{0.3}$ =31.5 kN/mm^2

Effective modulus of elasticity $E_{c,eff} = E_{cm} / (1+\varphi)$ = 31.5 / (1+2.85) = 8.15 kN/mm^2

Modular ratio for long term effect $\alpha_e = E_s / E_{c,eff}$ = 200 / 8.15 = 24.46

Check if section cracked
The uncracked neutral axis depth can be computed from the expression below (Bond et al. 2009)

$$x_u = \frac{bh^2/2 + (\alpha_e - 1)(A_s d + A_{s2} d_2)}{bh + (\alpha_e - 1)(A_s + A_{s2})}$$

as this is singly reinforced section, A_{s2} = 0

$$x_u = \frac{260 \times 500^2/2 + (24.46 - 1)(1296 \times 440)}{260 \times 500 + (24.46 - 1)(1296)} = 286 \text{ mm}$$

The uncracked second moment of area

$$I_u = \frac{bh^3}{12} + bh(h/2 - x_u)^2 + (\alpha_e - 1)[A_s(d - x)^2 + A_{s2}(x - d_2)^2]$$

$$I_u = \frac{260 \times 500^3}{12} + 260 \times 500(500/2 - 286)^2 + (24.46 - 1) \times 1296(440 - 286)^2$$

$$I_u = 3.598 \times 10^9\ mm^2$$

Cracking moment

$M_{cr} = f_{ct} I_u/(h-x)$ = 2.56 ×3.598 × 10^9/ (500-286) / 10^6

M_{cr}= 43.13 kNm < M_{qp} = 129.50 kNm → section cracked

M_{cr} = 43.13 kNm < M_{qp} = 129.50 kNm → section cracked

The crack with W_k may be calculated from Expression (7.8) of EC2:

$W_k = S_{r,max} (\varepsilon_{sm} - \varepsilon_{cm})$

The maximum crack spacing $S_{r,max}$ shall not exceed the values determined by Expression (7.11) and (7.14) of EC2, which are

- from Expression (7.11) $S_{r,max} = k_3c + k_1k_2k_4\phi/ \rho_{\rho,eff}$

For high bond bar, k_1 = 0.8, for bending calculation k_2 = 0.5, k_3 = 3.4 and k_4 = 0.425 as recommended by EC2

Since there are two diameters of rebar, 2ϕ25 + 1ϕ20, the diameter ϕ shall be taken as the equivalent diameter ϕ_{eq} (EC2 Eqn 7.12) and is computed as

$$\phi_{eq} = \frac{n_1\phi_1^2 + n_2\phi_2^2}{n_1\phi_1 + n_2\phi_2} = \frac{2 \times 25^2 + 20^2}{2 \times 25 + 20} = 23.6 \text{ mm}$$

For RC beam the effective ratio $\rho_{\rho,eff}$ is computed as

$$\rho_{\rho,eff} = A_s / A_{c,eff}$$

$A_{c,eff}$ is the effective area of concrete in tension surrounding the reinforcement of depth $h_{c,eff}$ where $h_{c,eff}$ is the lesser of 2.5(h-d), (h-x)/3, h/2.

The cracked neutral axis depth x can be determined from the Expression given in Bond et. al. (2009) as

$$x = \{[(A_s\alpha_e + A_{s2}(\alpha_e - 1))^2 + 2b(A_s d\alpha_e + A_{s2}d_2(\alpha_e - 1))]^{0.5} - (A_s\alpha_e + A_{s2}(\alpha_e - 1))\}/b$$

for singly reinforced section A_{s2} = 0, thus

$$x = \{[(1296 \times 24.46)^2 + 2 \times 260 \times 1296 \times 440 \times 24.46]^{0.5} - 1296 \times 24.46\}/260$$

$$x = 227.6\ mm$$

(h - x)/3 = (500-227.6)/3 = 90.8 mm < 2.5(h-d) = 150mm < h/2 = 250 mm

thus $h_{c,eff}$ = 90.8 mm

$A_{c,eff} = bh_{c,eff} - A_s$ = 260×90.8 - 1296 = 22313 mm^2

$\rho_{\rho,eff}$ = 1296 /22313 = 0.0581

$S_{r,max}$ computed from Expression (7.11) of EC2

$S_{r,max}$ = 3.4×47.5 + 0.8×0.5×0.425×23.6/0.0581 = 230.6 mm

$S_{r,max}$ computed from Expression (7.14) of EC2

$S_{r,max}$ = 1.3 (h-x) = 1.3 ×(500-227.6) = 354.1 mm

Therefore final maximum crack spacing $S_{r,max}$ = 230.6 mm

The average strain for crack width calculation can be determined from Expression (7.9) of EC2

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{\rho,eff}}\left(1 + \alpha_e \rho_{\rho,eff}\right)}{E_s} \geq 0.6 \frac{\sigma_s}{E_s}$$

σ_S is the stress in the tension reinforcement of a cracked section , which can be determined from the force equilibrium for section as shown in Figure 2 as follows:

Determine the concrete stress from moment equilibrium

$\sigma_C = M/[bx(d\text{-}x/3)/2]$

$\sigma_C = 129.50 \times 10^6 / [260\times227.6\ (440\text{-}227.6/3)/2]$

$\sigma_C = 12.02$ N/mm^2

Stress in tension steel

$\sigma_S = \sigma_C \cdot \alpha_e(d\text{-}x)/x = 12.32\times24.46\ (440\text{-}227.6)/227.6$

$\sigma_S = 274.4$ N/mm^2

For long term loading, the factor k_t = 0.4, therefore

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{274.4 - 0.4 \frac{2.56}{0.0581}(1 + 24.46 \times 0.0581)}{200000} = 0.001158 \geq 0.6 \frac{\sigma_s}{E_s} = 0.000818$$

The crack width is

$W_k = S_{r,max}\ (\varepsilon_{sm} - \varepsilon_{cm})$ = 230.6×0.001158 = 0.27 mm

This crack width is smaller than allowable crack width 0.30 mm, thus the provided reinforcement is acceptable.

Case 2: using $f_{yk} = 600$ N/mm^2 the required rebar area from strength calculation done earlier is $A_s = 951$ mm^2, choose 2ϕ25 ($A_{s,prov}$ = 982 mm^2) with bar spacing at 140mm.

By inspection as the A_s in this case is smaller than in Case 1 thus section is cracked under the same M_{qp} = 129.5 kNm.

Calculate neutral axis depth for cracked section

$$x = \{[(982 \times 24.46)^2 + 2 \times 260 \times 982 \times 440 \times 24.46]^{0.5} - 982 \times 24.46\}/260$$

$$x = 207.3\ mm$$

$(h\text{-}x)/3 = (500\text{-}207.3)/3 = 97.56$ mm < $2.5(h\text{-}d)$ = 150mm < $h/2$ = 250 mm

thus $h_{c,eff}$ = 97.56 mm

$A_{c,eff} = bh_{c,eff} - A_s$ = 260×97.56 -982 = 24383 mm^2

$\rho_{\rho,eff}$ = 982 /24383 = 0.0403

$S_{r,max}$ computed from Expression (7.11) of EC2

$S_{r,max} = k_3c + k_1k_2k_4\phi/\ \rho_{\rho,eff}$ = 3.4×48+0.8×0.5×0.425×25/0.0403 = 267.1 mm

$S_{r,max}$ computed from Expression (7.14) of EC2

$S_{r,max} = 1.3\ (h\text{-}x)$ = 1.3 ×(500-207.3) = 380.5 mm

Therefore final maximum crack spacing $S_{r,max}$ = 267.1 mm

Determine the concrete stress from moment equilibrium

$\sigma_C = M/[bx(d\text{-}x/3)/2]$

σ_c = 129.50 × 10^6 / [260×207.3(440-207.3/3)/2]

σ_c = 12.95 N/mm^2

Stress in tension steel

$\sigma_s = \sigma_c \cdot \alpha_e (d-x)/x$ = 12.95×24.46 (440-207.3)/ 207.3

σ_s = 355.7 N/mm^2

For long term loading, the factor k_t = 0.4, therefore

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{355.7 - 0.4\frac{2.56}{0.0403}(1 + 24.46 \times 0.0403)}{200000} = 0.001525 \geq 0.6\frac{\sigma_s}{E_s} = 0.001067$$

The crack width is

$W_k = S_{r,max}(\varepsilon_{sm} - \varepsilon_{cm})$ = 267.1×0.001525 = 0.41 mm

This crack width exceeds the allowable crack width of 0.30 mm, thus the provided reinforcement is not sufficient for serviceability design requirement. The provided reinforcement has to be increased for example, by adding one ϕ20 rebar, i.e similar reinforcement as in case 1, we will get the same crack width in case 1 of 0.27mm that can satisfy the crack width requirement.

Comment: In this case when we change reinforcement from grade B500 to grade B600, we may reduce the reinforcement area based on strength calculation but after checking the serviceability design requirement the reduced reinforcement area cannot satisfy the crack width requirement, therefore similar area of reinforcement as for the case of grade B500 is needed.

Case 3: Similar with case 2 but the moment under the action of the quasi-permanent load combination is $M_{qp} = 0.51M$ =94.35 kNm, the ratio 0.51 corresponds with the case dead load equal to 1.5 live load and M = 1.35 DL + 1.5 LL, M_{qp} = 1.0 DL + 0.3 LL. Using f_{yk} = 600 N/mm^2 the required rebar area from strength calculation done earlier is A_s = 951 mm^2, choose 2ϕ25 ($A_{s,prov}$ = 982 mm^2) with bar spacing at 140mm.

Check if section cracked
The uncracked neutral axis depth can be computed similar to case 1 with A_s = 982 mm^2 as below

$$x_u = \frac{260 \times 500^2/2 + (24.46 - 1)(982 \times 440)}{260 \times 500 + (24.46 - 1)(982)} = 278.6 \text{ mm}$$

The uncracked second moment of area for singly reinforced section

$$I_u = \frac{bh^3}{12} + bh(h/2 - x_u)^2 + (\alpha_e - 1)A_s(d - x)^2$$

$$I_u = \frac{260 \times 500^3}{12} + 260 \times 500(500/2 - 278.6)^2 + (24.46 - 1) \times 982(440 - 278.6)^2$$

$$I_u = 3.415 \times 10^9 \; mm^2$$

Cracking moment

$M_{cr} = f_{ct}I_u/(h-x)$ = 2.56 ×3.415 × 10^9/ (500-279) / 10^6

M_{cr}= 39.56 kNm < M_{qp} = 94.35 kNm → section cracked

Calculate neutral axis depth for cracked section

$$x = \{[(982 \times 24.46)^2 + 2 \times 260 \times 982 \times 440 \times 24.46]^{0.5} - 982 \times 24.46\}/260$$

$$x = 207.3\ mm$$

(h-x)/3 = (500-207.3)/3 = 97.56 mm < 2.5(h-d) = 150mm < h/2 = 250 mm

thus $h_{c,eff}$ = 97.56 mm

$A_{c,eff} = bh_{c,eff} - A_s$ = 260×97.56 -982 = 24383 mm^2

$\rho_{\rho,eff}$ = 982 /24383 = 0.0403

$S_{r,max}$ computed from Expression (7.11) of EC2

$S_{r,max} = k_3c + k_1k_2k_4\phi/\rho_{\rho,eff}$ = 3.4×48+0.8×0.5×0.425×25/0.0403 = 267.1 mm

$S_{r,max}$ computed from Expression (7.14) of EC2

$S_{r,max}$ = 1.3 (h-x) = 1.3 ×(500-207.3) = 380.5 mm

Therefore final maximum crack spacing $S_{r,max}$ = 267.1 mm

Determine the concrete stress from moment equilibrium

$\sigma_c = M/[bx(d-x/3)/2]$

σ_c = 94.35 × 10^6 / [260×207.3(440-207.3/3)/2]

σ_c = 9.44 N/mm^2

Stress in tension steel

$\sigma_s = \sigma_c \cdot \alpha_e(d-x)/x$ = 9.44×24.46 (440-207.3)/ 207.3

σ_s = 259.1 N/mm^2

For long term loading, the factor k_t = 0.4, therefore

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{259.1 - 0.4\frac{2.56}{0.0403}(1 + 24.46 \times 0.0403)}{200000} = 0.001042 \geq 0.6\frac{\sigma_s}{E_s} = 0.000777$$

The crack width is

$W_k = S_{r,max}(\varepsilon_{sm} - \varepsilon_{cm})$ = 267.1×0.001042 = 0.28 mm

This crack width is smaller than allowable crack width 0.30 mm, thus the provided reinforcement of 2ϕ25 is acceptable.

The results of case 2 and case 3 above illustrate that when using high strength rebar grade 600, the designer needs to check the serviceablity limit state as the reduction of reinforcement area may lead to excessive crack width.

Example 1.2: Flexural design of doubly reinforced beam

Determine the areas of reinforcement required for the section shown in Figure 3 to resist an ultimate design moment of 285 kNm. The characteristic strengths are f_{yk} = 600 N/mm^2 for the reinforcement and f_{ck} = 25 N/mm^2 for the concrete.

Note: This example is similar to Example 4.3 in the text book by Mosley et al "Reinforced Concrete Design to EC2" but using high strength steel instead of f_{yk} = 500 N/mm^2 in the Example 4.3.

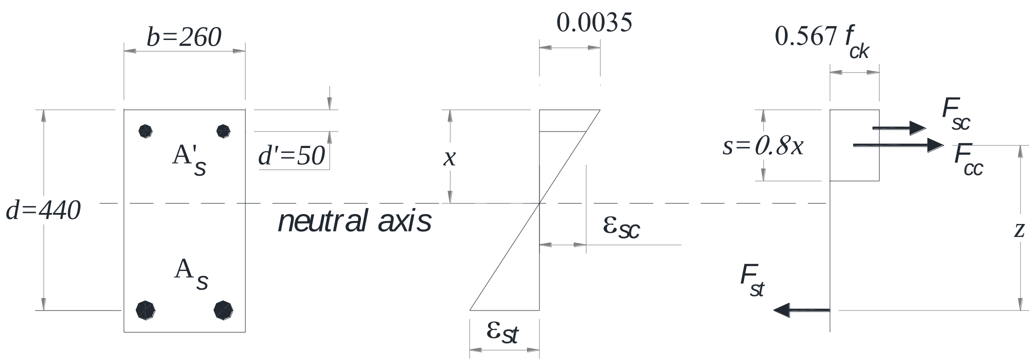


Figure 3 Section in example 1.2

Calculate $K = M/bd^2f_{ck} = 285 \times 10^6 / (260\times440^2\times25) = 0.226 > 0.167$

therefore compression steel is required.

Since *d'/d* = 50/440 = 0.11 < 0.117 thus compression steel will have yielded.

Compression steel area

$$A_s' = \frac{(K - K_{bal})f_{ck}bd^2}{0.87f_{yk}(d-d')} = \frac{(0.226-0.167)25\times260\times440^2}{0.87\times600(440-50)} = 365 \text{ mm}^2$$

Tension steel area

$$A_s = \frac{K_{bal}f_{ck}bd^2}{0.87f_{yk}z_{bal}} + A_s' = \frac{0.167\times25\times260\times440^2}{0.87\times600(0.82\times440)} + 365 = 1116 + 365 = 1481 \text{ mm}^2$$

Comment: Example 4.3 of the text book that uses f_{yk} = 500 N/mm2 gives A'$_s$ = 438 mm^2 and A_s = 1777 mm^2. Therefore a saving of 17% in total steel area is achieved for this doubly reinforced section when changing from Grade B500 to Grade B600 steel. However once the service load is known, further checking of crack and deflection should be carried out to ensure the rebar provided can satisfy both strength and serviceability requirements.

2. Design chart for columns

The design charts for rectangular columns have been prepared for concrete strengths of C30/37 and C90/105. The value of *d/h* have been chosen between 0.75 and 0.9 in steps of 0.05. Reinforcement ratios have been taken from 1.0 to 8% in steps of 1%. The centre line of the reinforcement is shown on a diagram of each graph. It is assumed that all the reinforcement is distributed symmetrically over the length of the two centre lines.

For all charts f_{yk} is taken as 600 N/mm^2 and the partial safety factor for reinforcement is γ_m = 1.15. Modulus of elasticity of reinforcement steel has been taken as 200,000 N/mm^2.

The parabolic-rectangle stress block was used for concrete stress distribution.

Chart 2.1 f_{ck} = 30 N/mm^2 d/h = 0.75

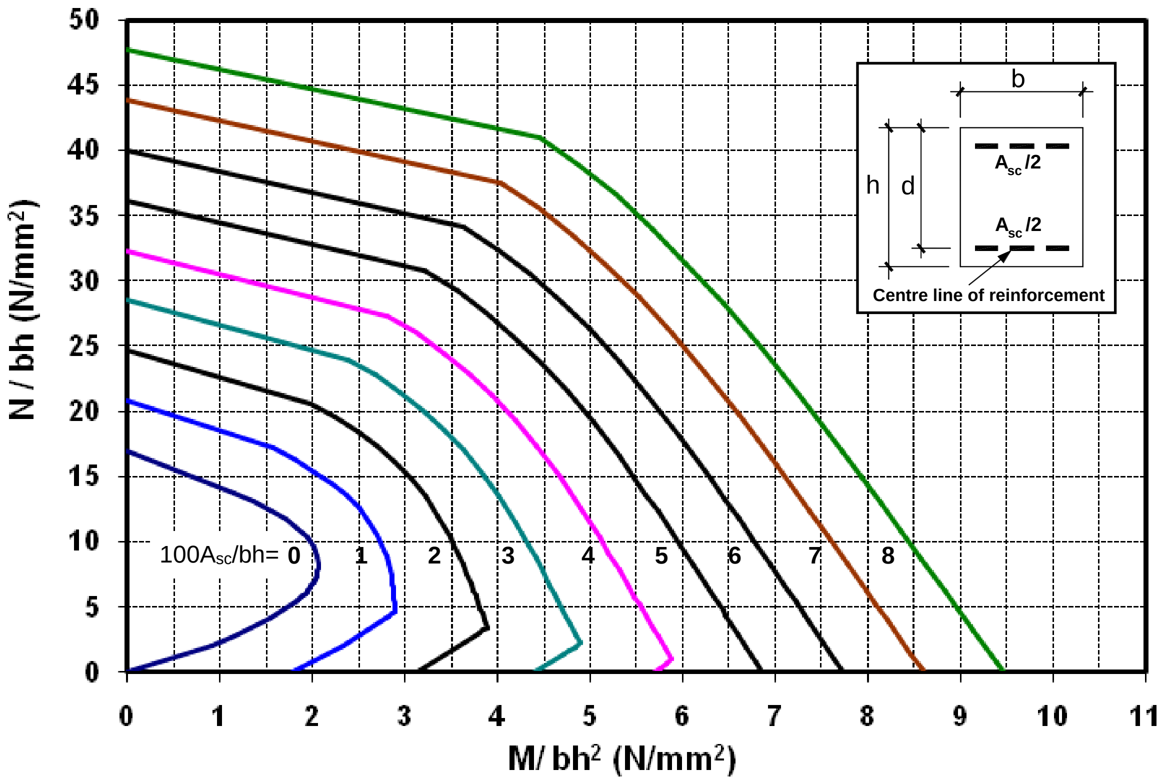


Chart 2.2 f_{ck} = 30 N/mm^2 d/h = 0.80

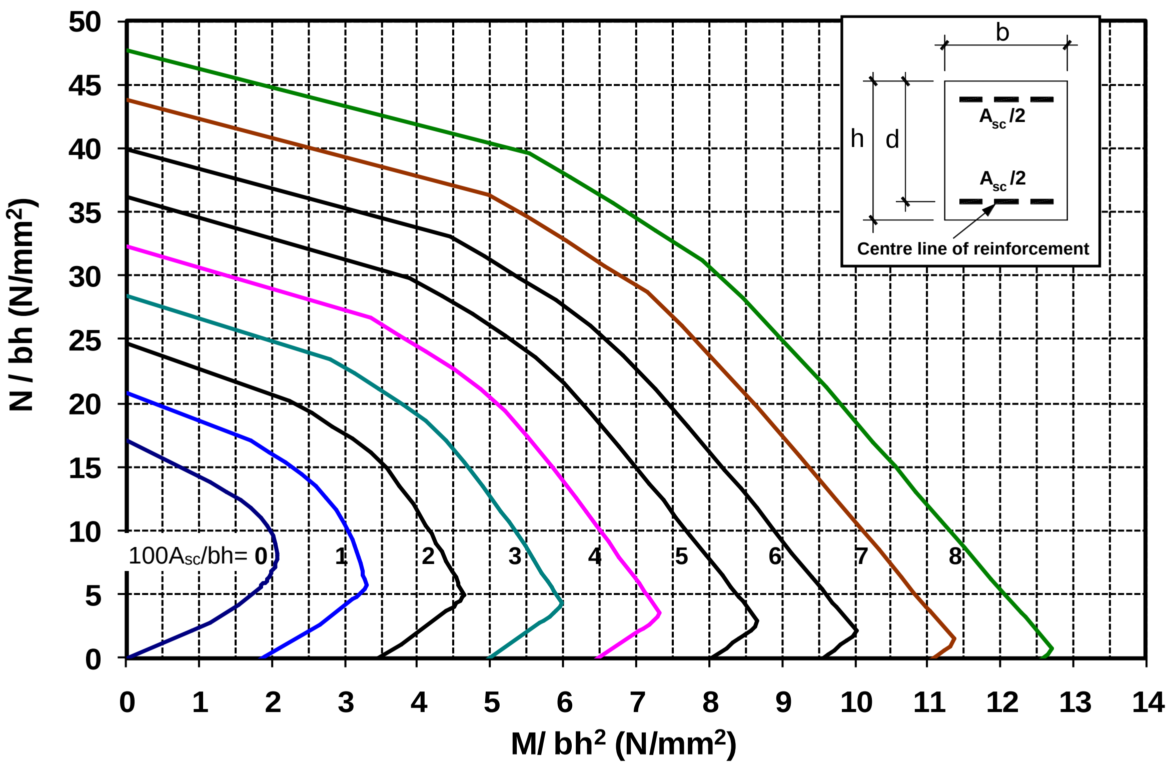


Chart 2.3 f_{ck}= 30 N/mm^2 d/h = 0.85

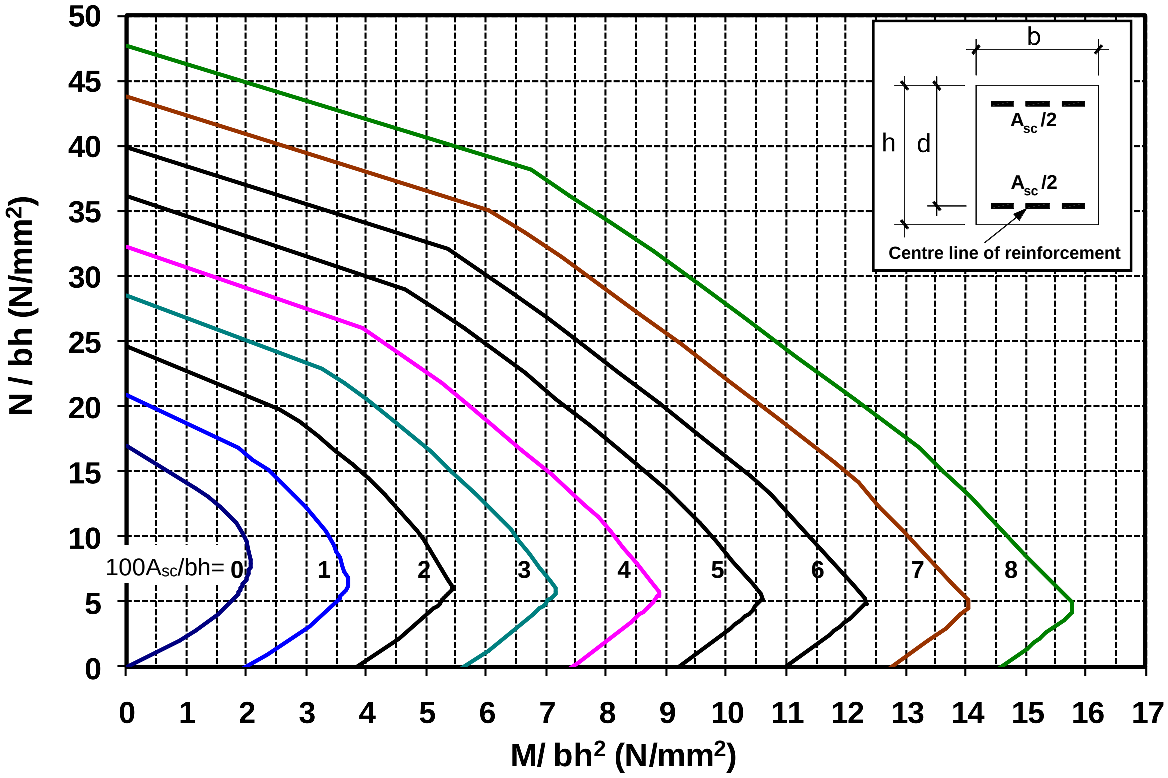


Chart 2.4 f_{ck} = 30 N/mm^2 d/h = 0.90

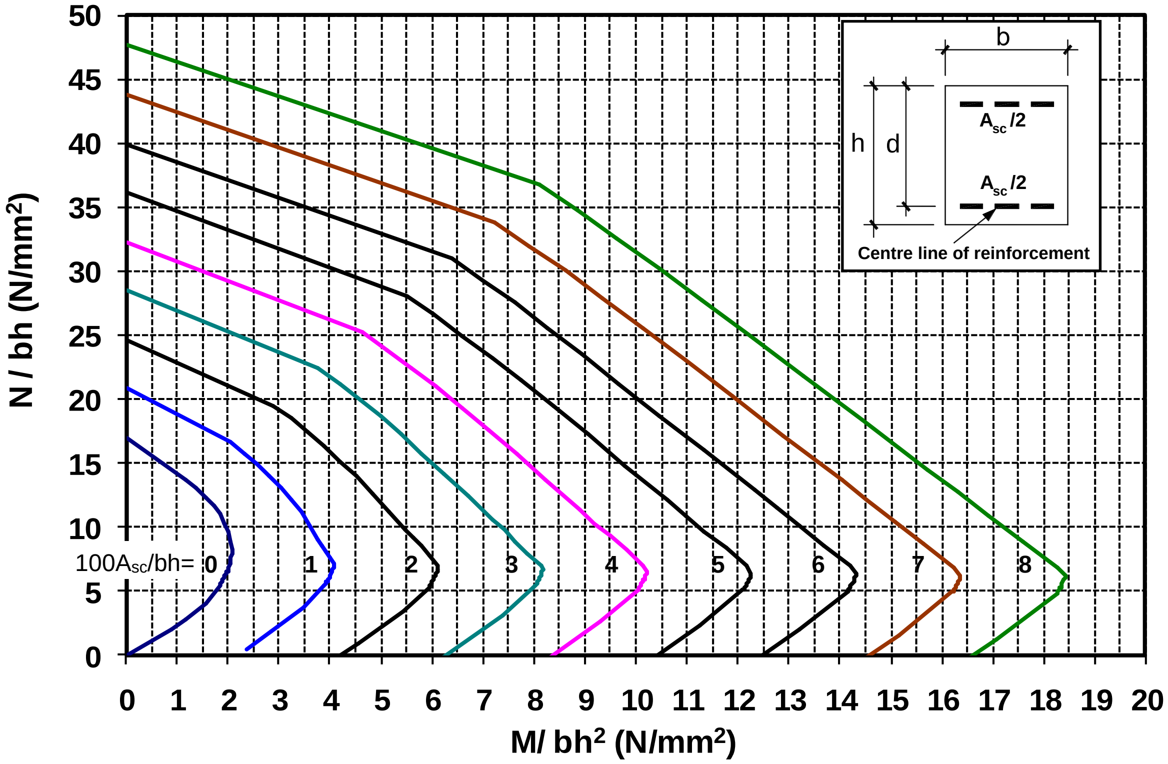


Chart 2.5 f_{ck}= 90 N/mm^2 d/h = 0.75

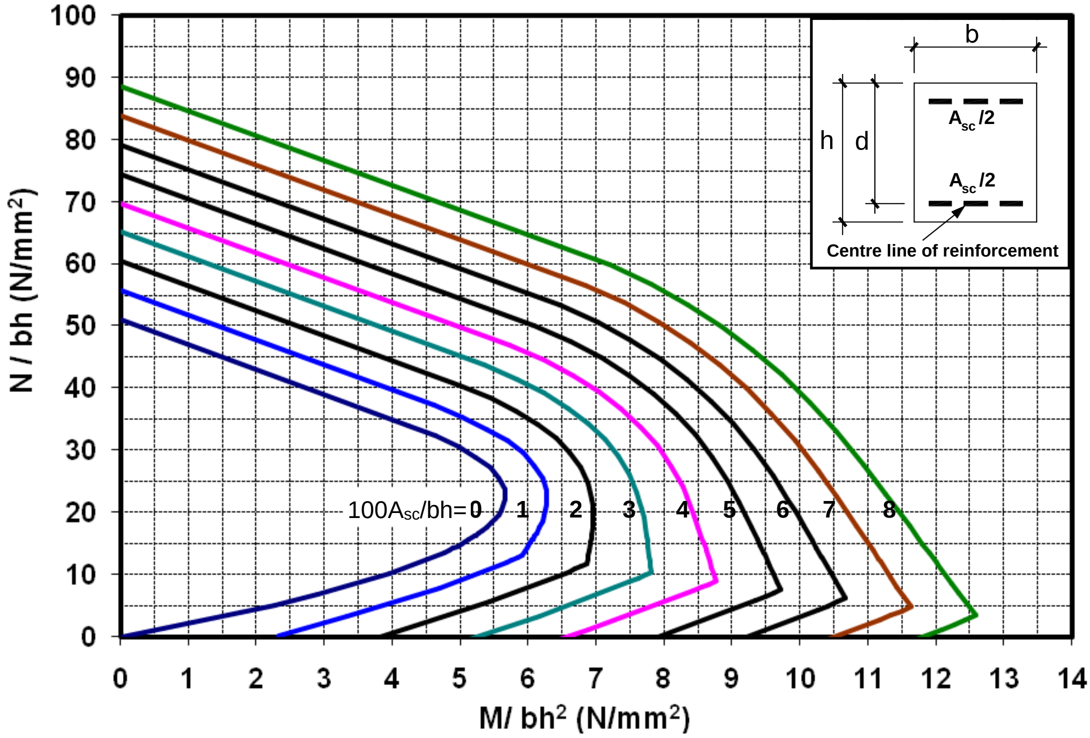


Chart 2.6 f_{ck}= 90 N/mm^2 d/h = 0.80

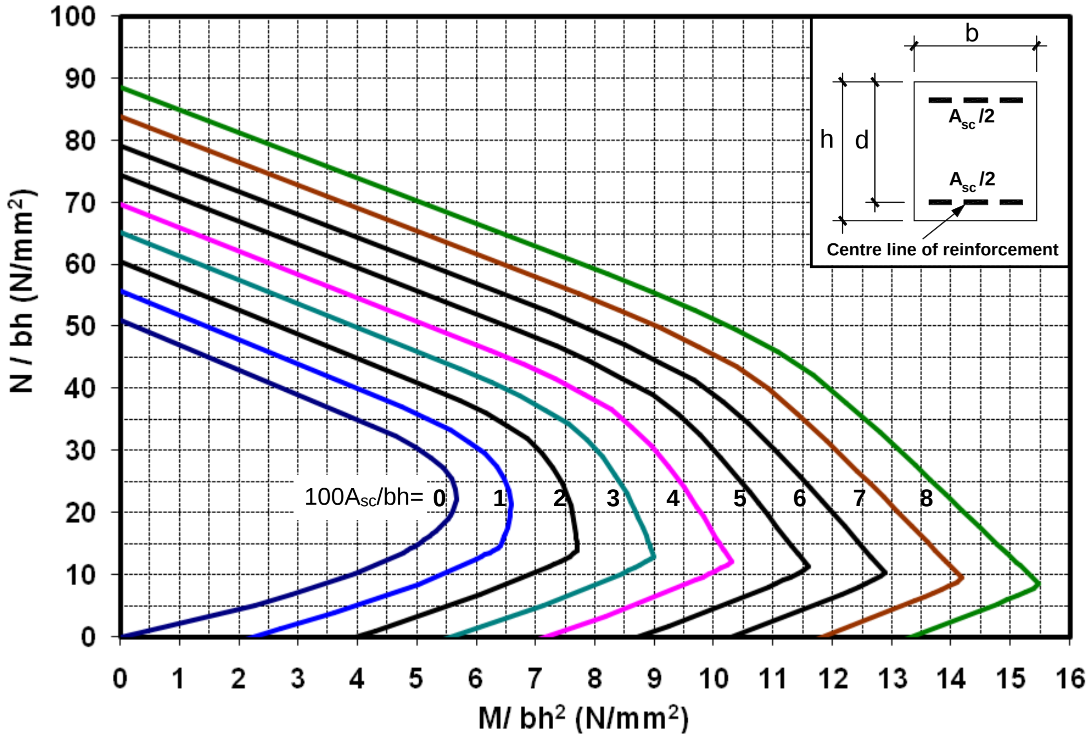


Chart 2.7 $f_{ck} = 90$ N/mm^2 $d/h = 0.85$

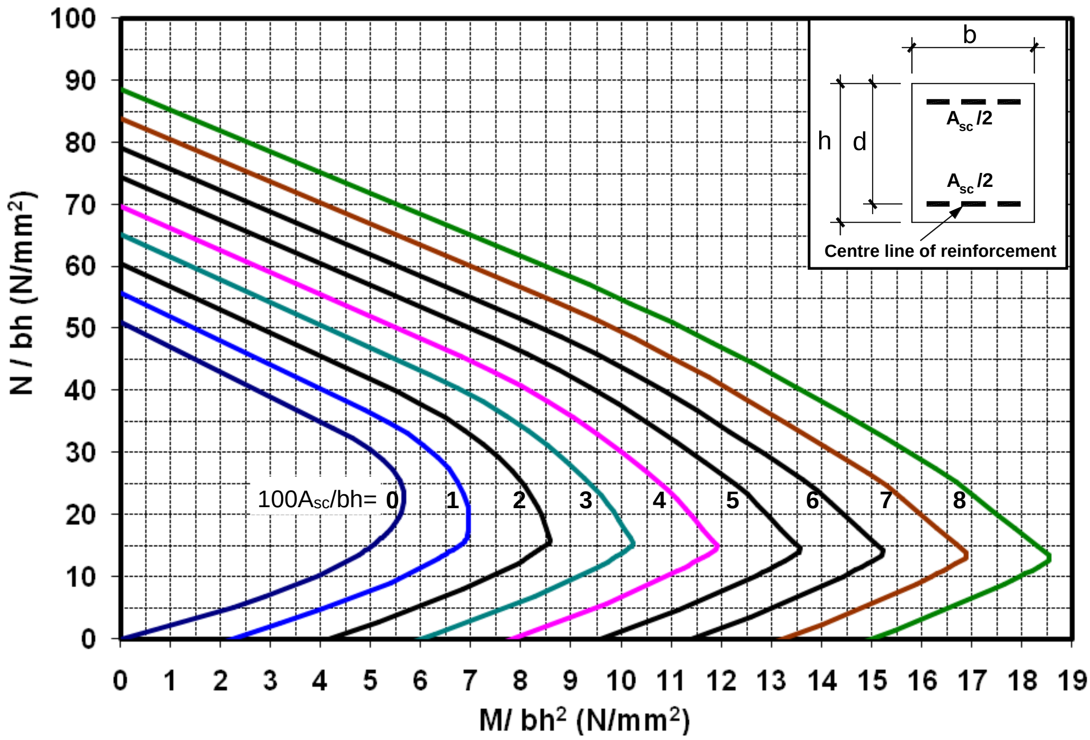
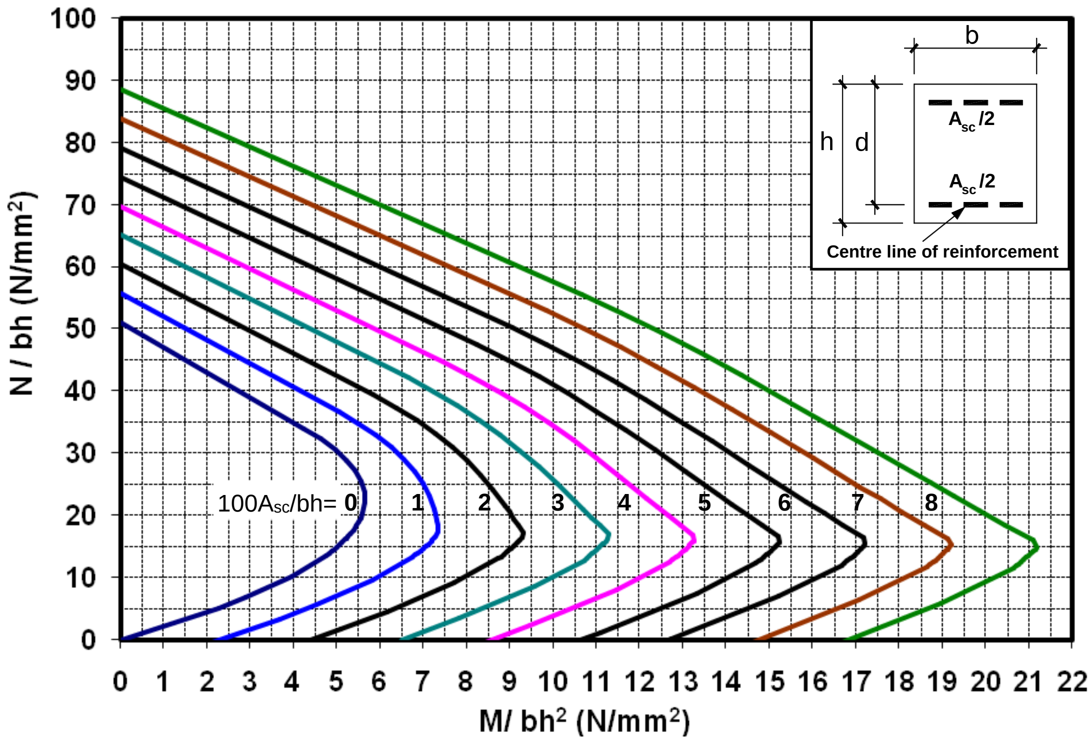


Chart 2.8 $f_{ck} = 90$ N/mm^2 $d/h = 0.90$



3. Moment redistribution for beams

Example 3.1 Design of tension and compression reinforcement with concrete strength f_{ck} = 25 MPa.

For the beam shown in Figure 4, the ultimate moment before redistribution is 462.5 kNm (hogging moment at support) of the beam. For comparison purpose, different cases are assumed:

- Case 1: No moment redistribution, rebar grade B500B to SS 560:2016;

- Case 2: No moment redistribution, rebar grade B600B to SS 560:2016;

- Case 3: 20% moment redistribution, rebar grade B500B to SS 560:2016, using Singapore Annex to EC2;

- Case 4: 20% moment redistribution, rebar grade B600B to SS 560:2016, using Singapore Annex to EC2;

- Case 5: 20% moment redistribution, rebar grade B500B to SS 560:2016, using recommended values in EC2;

- Case 6: 20% moment redistribution, rebar grade B600B to SS 560:2016, using recommended values in EC2.

For the cases of 20% moment redistribution with ultimate moment reduced to 370 MPa, it is assumed that the additional sagging moment redistributed to mid span would be recalculated elsewhere.

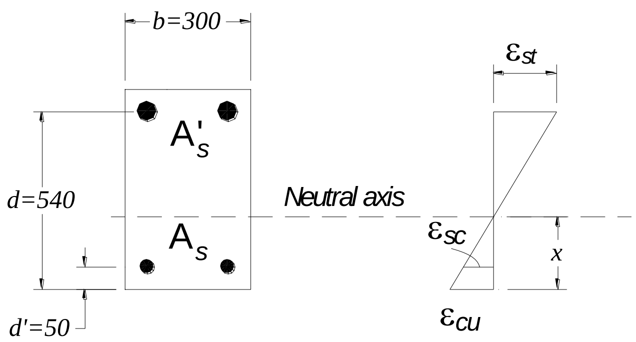


Figure 4 Beam doubly reinforced to resist a hogging moment

Case 1: No moment redistribution, rebar grade B500B to SS 560:2016

Calculate parameter K

$$K = M / bd^2 f_{ck} = 462.5\times10^6 / 300\times540^2\times25 = 0.211$$

Without moment redistribution, from *Table 4-4* we have K_{bal} = 0.167, $K > K_{bal}$ thus compression reinforcement is needed.

The neutral axis depth $\quad x = 0.45d = 0.45\times 540 = 243$ mm

Steel compressive strain:

$$\varepsilon_{sc} = \frac{0.0035(x-d')}{x} = \frac{0.0035(243-50)}{243} = 0.00278 > 0.00217$$

thus compression steel has yielded

Required compression steel area:

$$A'_s = \frac{(K-K_{bal})f_{ck}bd^2}{0.87f_{yk}(d-d')} = \frac{(0.211-0.167)\times 25\times 300\times 540^2}{0.87\times 500\times(540-50)} = 456 \text{ mm}^2$$

Try 2ϕ20 for A'_s, area = 628 mm^2

The tension steel area

$$A_s = \frac{K_{bal}f_{ck}bd^2}{0.87f_{yk}z} + A'_s$$

where z = d-0.4x = 540 - 0.4$\times$243 = 443 mm

$$A_s = \frac{0.167\times 25\times 300\times 540^2}{0.87\times 500\times 443} + 456 = 2352 \text{ mm}^2$$

Try 5 ϕ25 for A_s, area = 2454 mm^2

Check

$(A'_{s,prov} - A'_{s,req})$ = 628 - 456= 172 mm^2

$(A_{s,prov} - A_{s,req})$ = 2454 - 2352 = 102 mm^2

$(A'_{s,prov} - A'_{s,req})$ > $(A_{s,prov} - A_{s,req})$ => ductility is ensured

<u>Case 2</u>: No moment redistribution, rebar grade B600B to SS 560:2016

Similar to case 1 with the limitation x/d = 0.45 we will have *K* = 0.211 > K_{bal} = 0.167 and compression reinforcement is needed.

The neutral axis depth $x = 0.45d = 0.45\times 540 = 243$ mm

Steel compressive strain:

$$\varepsilon_{sc} = \frac{0.0035(x-d')}{x} = \frac{0.0035(243-50)}{243} = 0.00278 > 0.00261$$

thus compression steel has yielded

Required compression steel area:

$$A'_s = \frac{(K-K_{bal})f_{ck}bd^2}{0.87f_{yk}(d-d')} = \frac{(0.211-0.167)\times 25\times 300\times 540^2}{0.87\times 600\times(540-50)} = 380 \text{ mm}^2$$

Try 2ϕ16 for A'_s, area = 402 mm^2

The tension steel area

$$A_s = \frac{K_{bal} f_{ck} b d^2}{0.87 f_{yk} z} + A'_s$$

where $z = d$-0.4x = 540 - 0.4$\times$243 = 443 mm

$$A_s = \frac{0.167 \times 25 \times 300 \times 540^2}{0.87 \times 600 \times 443} + 380 = 1960 \text{ mm}^2$$

Provide 4 ϕ25 for A_s, area = 1963 mm^2

Check

$(A'_{s,prov} - A'_{s,req})$ = 402 - 380 = 22 mm^2

$(A_{s,prov} - A_{s,req})$ = 1963 - 1960 = 3 mm^2

$(A'_{s,prov} - A'_{s,req})$ > $(A_{s,prov} - A_{s,req})$ => ductility is ensured.

Case 3: Design for 20% moment redistribution, rebar grade B500B to SS 560:2016, using Singapore Annex to EC2

The reduced ultimate moment M = 370 kNm

Using Singapore Annex to EC2 we have parameters k_1 = 0.4, k_2 = 1.0

Calculate parameter *K*

$K = M / bd^2 f_{ck}$ = 370×10^6 / 300×540^2×25 = 0.169

With 20% redistribution, from *Table 4-4* we have x_u/d = 0.4 and K_{bal} = 0.152

$K > K_{bal}$ thus compression reinforcement is needed.

The neutral axis depth $x = 0.4d$ = 0.4× 540 = 216 mm

Steel compressive strain:

$$\varepsilon_{sc} = \frac{0.0035(x - d')}{x} = \frac{0.0035(216 - 50)}{216} = 0.00269 > 0.00217$$

thus compression steel has yielded

Required compression steel area:

$$A'_s = \frac{(K - K_{bal}) f_{ck} b d^2}{0.87 f_{yk} (d - d')} = \frac{(0.169 - 0.152) \times 25 \times 300 \times 540^2}{0.87 \times 500 \times (540 - 50)} = 173 \text{ mm}^2$$

Try 2ϕ16 for A'_s, area = 402 mm^2

The tension steel area

$$A_s = \frac{K_{bal} f_{ck} b d^2}{0.87 f_{yk} z} + A'_s$$

where $z = d$-0.4x = 540 - 0.4$\times$216 = 454 mm

$$A_s = \frac{0.152 \times 25 \times 300 \times 540^2}{0.87 \times 500 \times 454} + 173 = 1861 \text{ mm}^2$$

Try 4 ϕ25 for A_s, area = 1963 mm^2

Check

$(A'_{s,prov} - A'_{s,req})$ = 402 – 173 = 229 mm^2

$(A_{s,prov} - A_{s,req})$ = 1963-1861 = 102 mm^2

$(A'_{s,prov} - A'_{s,req})$ > $(A_{s,prov} - A_{s,req})$ => ductility is ensured

<u>Case 4:</u> Design for 20% moment redistribution, rebar grade B600B to SS 560:2016, using Singapore Annex to EC2

The reduced ultimate moment M = 370 kNm

Using Singapore Annex to EC2 we have parameters k_1 = 0.4, k_2 = 1.0

Calculate parameter K

$K = M / bd^2 f_{ck}$ = 370×10^6 / 300×540^2×25 = 0.169

With 20% redistribution, from *Table 4-4* we have x_u/d = 0.4 and K_{bal} = 0.152

$K > K_{bal}$ thus compression reinforcement is needed.

The neutral axis depth $x = 0.4d$ = 0.4× 540 = 216 mm

Steel compressive strain:

$$\varepsilon_{sc} = \frac{0.0035(x - d')}{x} = \frac{0.0035(216 - 50)}{216} = 0.00269 > 0.00261$$

thus compression steel has yielded

Required compression steel area:

$$A'_s = \frac{(K - K_{bal}) f_{ck} b d^2}{0.87 f_{yk}(d - d')} = \frac{(0.169 - 0.152) \times 25 \times 300 \times 540^2}{0.87 \times 600 \times (540 - 50)} = 144 \text{ mm}^2$$

Try 2ϕ16 for A'_s, area = 402 mm^2

The tension steel area

$$A_s = \frac{K_{bal} f_{ck} b d^2}{0.87 f_{yk} z} + A'_s$$

where $z = d - 0.4x$ = 540 - 0.4× 216 = 454 mm

$$A_s = \frac{0.152 \times 25 \times 300 \times 540^2}{0.87 \times 600 \times 454} + 144 = 1551 \text{ mm}^2$$

Provide 2 ϕ32 for A_s, area = 1608 mm^2

Check

$(A'_{s,prov} - A'_{s,req})$ = 402 - 144 = 258 mm^2

$(A_{s,prov} - A_{s,req})$ = 1608 - 1551 = 57 mm^2

$(A'_{s,prov} - A'_{s,req})$ > $(A_{s,prov} - A_{s,req})$ => ductility is ensured

<u>**Case 5:**</u> Design for 20% moment redistribution, rebar grade B500B to SS 560:2016, using recommended values of EC2

The reduced ultimate moment M = 370 kNm

Using recommended values of EC2 we have parameters k_1 = 0.44, k_2 = 1.25

With 20% redistribution, from *Table 4-4* we have x_u/d = 0.288 and K_{bal} = 0.116

K = M / bd^2f_{ck} = 370×10^6 / 300×540^2×25 = 0.169 > K_{bal} thus compression reinforcement is needed.

The neutral axis depth x = 0.288d = 0.288× 540 = 156 mm

Steel compressive strain:

$$\varepsilon_{sc} = \frac{0.0035(x - d')}{x} = \frac{0.0035(156 - 50)}{156} = 0.00238 > 0.00217$$

thus compression steel has yielded

Compression steel area:

$$A'_s = \frac{(K - K_{bal})f_{ck}bd^2}{0.87f_{yk}(d - d')} = \frac{(0.169 - 0.116) \times 25 \times 300 \times 540^2}{0.87 \times 500 \times (540 - 50)} = 551 \text{ mm}^2$$

Provide 2ϕ20+1ϕ16 for A'_s, area = 829 mm^2

The tension steel area

$$A_s = \frac{K_{bal}f_{ck}bd^2}{0.87f_{yk}z} + A'_s$$

where $z = d$-0.4x = 540 - 0.4× 156 = 478 mm

$$A_s = \frac{0.116 \times 25 \times 300 \times 540^2}{0.87 \times 500 \times 478} + 551 = 1766 \text{ mm}^2$$

Provide 4 ϕ25 for A_s, area = 1963 mm^2

Check

$(A'_{s,prov} - A'_{s,req})$ = 829 - 551 = 278 mm^2

$(A_{s,prov} - A_{s,req})$ = 1963 - 1766 = 197 mm^2

$(A'_{s,prov} - A'_{s,req})$ > $(A_{s,prov} - A_{s,req})$ => ductility is ensured

Case 6: Design for 20% moment redistribution, rebar grade B600B to SS 560:2016, using recommended values of EC2

The reduced ultimate moment M = 370 kNm

Using recommended values of EC2 we have parameters k_1 = 0.44, k_2 = 1.25

With 20% redistribution, from *Table 4-4* we have x_u/d = 0.288 and K_{bal} = 0.116

$K = M / bd^2 f_{ck}$ = 370×10^6 / 300×540^2×25 = 0.169 > K_{bal} thus compression reinforcement is needed.

The neutral axis depth x = 0.288d = 0.288× 540 = 156 mm

Steel compressive strain:

$$\varepsilon_{sc} = \frac{0.0035(x - d')}{x} = \frac{0.0035(156 - 50)}{156} = 0.00238 < 0.00261$$

thus $f_{sc} < 0.87 f_{yk}$

steel compressive stress, $f_{sc} = E_s \varepsilon_{sc}$ = 200 000 × 0.00238 = 475 N/mm^2

Compression steel area:

$$A'_s = \frac{(K - K_{bal}) f_{ck} b d^2}{f_{sc}(d - d')} = \frac{(0.169 - 0.116)\times 25 \times 300 \times 540^2}{475 \times (540 - 50)} = 504 \text{ mm}^2$$

Provide 2ϕ20 for A'_s, area = 628 mm^2

The tension steel area

$$A_s = \frac{K_{bal} f_{ck} b d^2}{0.87 f_{yk} z} + A'_s \frac{f_{sc}}{0.87 f_{yk}}$$

where $z = d$-0.4x = 540 - 0.4 × 156 = 478 mm

$$A_s = \frac{0.116 \times 25 \times 300 \times 540^2}{0.87 \times 600 \times 478} + 504 \frac{475}{0.87 \times 600} = 1472 \text{ mm}^2$$

Provide 3 ϕ25 for A_s, area = 1473 mm^2

Check

$(A'_{s,prov} - A'_{s,req})$ = 628 - 504 = 124 mm^2

$(A_{s,prov} - A_{s,req})$ = 1473 - 1472 = 1 mm^2

$(A'_{s,prov} - A'_{s,req})$ > $(A_{s,prov} - A_{s,req})$ => ductility is ensured

Table 1 Summary of required reinforcement area for the six cases of Example 3.1

Rebar	No moment redistribution		20% Moment redistribution			
			Singapore Annex		Recommended value EC2	
	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
	f_{yk} = 500 N/mm2	f_{yk} = 600 N/mm2	f_{yk} = 500 N/mm2	f_{yk} = 600 N/mm2	f_{yk} = 500 N/mm2	f_{yk} = 600 N/mm2
$A'_{s,req}$	456	380	173	144	551	504
$A_{s,req}$	2352	1960	1861	1551	1766	1472
$A'_{s,prov}$	628	402	402	402	829	628
$A_{s,prov}$	2454	1963	1963	1608	1963	1473

Comments : From Table 1 it can be seen that using parameters from EC2 requires more compression reinforcement than using those provided by Singapore NA for the same percentage of moment redistribution. With the same parameters, a section using higher strength reinforcement requires less reinforcement than the one using lower strength reinforcement. It is necessary to check that the extra amount of reinforcement provided for compression reinforcement compared to the required area shall not be less than that for tension reinforcement to ensure the ductility requirement.

Example 3.2 Design of tension and compression reinforcement for the beam with concrete strength f_{ck} = 60 MPa.

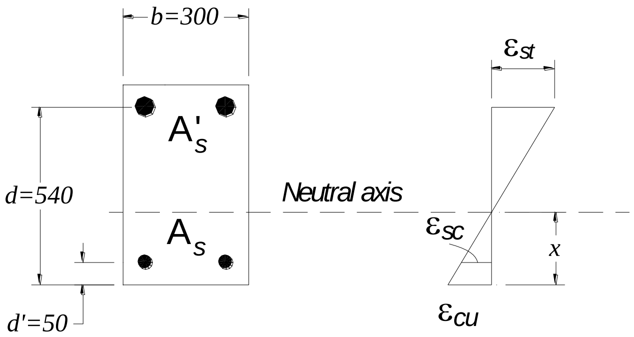


Figure 5 Beam section in Example 3-2 to resist a hogging moment

The ultimate moment before redistribution is 750 kNm causing hogging of the beam. For comparison purpose, different cases are assumed:

- Case 1: No moment redistribution, rebar grade B500B to SS 560:2016;

- Case 2: No moment redistribution, rebar grade B600B to SS 560:2016;

- Case 3: 20% moment redistribution, rebar grade B500B to SS 560:2016, using Singapore Annex to EC2;

- Case 4: 20% moment redistribution, rebar grade B600B to SS 560:2016, using Singapore Annex to EC2;

- Case 5: 20% moment redistribution, rebar grade B500B to SS 560:2016, using recommended values in EC2;

- Case 6: 20% moment redistribution, rebar grade B600B to SS 560:2016, using recommended values in EC2.

For the cases of 20% moment redistribution with ultimate moment reduced to 560 Mpa, it is assumed that the additional sagging moment redistributed to mid span would be recalculated elsewhere.

<u>Case 1</u>: No moment redistribution, rebar grade B500B to SS 560:2016

For high strength concrete f_{ck} = 60 N/mm^2 we have λ = 0.8- (60-50)/400 = 0.775, η=1-(60-50)/200 = 0.95, ε_{cu} = 0.00288

Calculate parameter K

$K = M / bd^2 f_{ck}$ = 700×10^6 / 300×540^2×60 = 0.133

Without moment redistribution, from *Table 4-5* for f_{ck} = 60 N/mm^2 and using Singapore Annex we have K_{bal} = 0.126*,* $K > K_{bal}$ thus compression reinforcement is needed.

The neutral axis depth $x = 0.35d$ = 0.35× 540 = 189 mm

Steel compressive strain:

$$\varepsilon_{sc} = \frac{\varepsilon_{cu}(x - d')}{x} = \frac{0.00288(189 - 50)}{189} = 0.00212 < 0.00217$$

thus $f_{sc} < 0.87 f_{yk}$

steel compressive stress: $f_{sc} = E_s \varepsilon_{sc}$ = 200 000 × 0.00212 = 424 N/mm^2

Required compression steel area:

$$A'_s = \frac{(K - K_{bal}) f_{ck} b d^2}{f_{sc}(d - d')} = \frac{(0.133 - 0.126) \times 60 \times 300 \times 540^2}{424 \times (540 - 50)} = 180 \text{ mm}^2$$

Try 2ϕ20 for A'_s, area = 628 mm^2

The tension steel area

$$A_s = \frac{K_{bal} f_{ck} b d^2}{0.87 f_{yk} z} + A'_s \frac{f_{sc}}{0.87 f_{yk}}$$

where $z = d - 0.5\lambda x$ = 540 - 0.5×0.775×189 = 467 mm

$$A_s = \frac{0.126 \times 60 \times 300 \times 540^2}{0.87 \times 500 \times 467} + 180 \frac{424}{0.87 \times 500} = 3439 \text{ mm}^2$$

Try 4 ϕ25 + 2 ϕ32 for A_s, area = 3572 mm^2

Check

$(A'_{s,prov} - A'_{s,req})$ = 628 - 180 = 443 mm^2

$(A_{s,prov} - A_{s,req})$ = 3572 - 3459 = 133 mm^2

$(A'_{s,prov} - A'_{s,req}) > (A_{s,prov} - A_{s,req})$ => ductility is ensured

Case 2: No moment redistribution, rebar grade B600B to SS 560:2016

Similar to case 1 with the limitation x/d = 0.35 we have K = 0.133 > K_{bal} = 0.126 and compression reinforcement is needed. Similar to case 1 we have x = 189 mm and ε_{sc} = 0.00212

Steel compressive stress: $f_{sc} = E_s\varepsilon_{sc}$ = 200 000 $\times$ 0.00212 = 424 N/mm^2

Required compression steel area:

$$A'_s = \frac{(K - K_{bal})f_{ck}bd^2}{f_{sc}(d - d')} = \frac{(0.133 - 0.126)\times 60\times 300\times 540^2}{424\times(540 - 50)} = 180 \text{ mm}^2$$

Try 2ϕ20 for A'_s, area = 628 mm^2

The tension steel area

$$A_s = \frac{K_{bal}f_{ck}bd^2}{0.87f_{yk}z} + A'_s\frac{f_{sc}}{0.87f_{yk}}$$

where $z = d$-0.5λx = 540 - 0.5$\times$0.775$\times$189 = 467 mm

$$A_s = \frac{0.126 \times 60 \times 300 \times 540^2}{0.87 \times 600 \times 467} + 180\frac{424}{0.87 \times 600} = 2866 \text{ mm}^2$$

Try 6 ϕ25 for A_s, area = 2945 mm^2

Check

$(A'_{s,prov} - A'_{s,req})$ = 628 - 180 = 448 mm^2

$(A_{s,prov} - A_{s,req})$ = 2945 - 2866 = 80 mm^2

$(A'_{s,prov} - A'_{s,req})$ > $(A_{s,prov} - A_{s,req})$ => ductility is ensured

Case 3: Design for 20% moment redistribution, rebar grade B500B to SS 560:2016, using Singapore Annex to EC2

The reduced ultimate moment M = 560 kNm

Using Singapore Annex to EC2 we have parameters k_3 = 0.4, $k_4 = 0.6+0.0014/\varepsilon_{cu}$ = 0.6+0.0014/0.00288 = 1.086

Calculate parameter K

$K = M / bd^2f_{ck}$ = 560×10^6 / 300×540^2×60 = 0.107

With 20% redistribution, from *Table 4-5* using Singapore Annex we have x_u/d = 0.35 and K_{bal} = 0.126

$K < K_{bal}$ thus compression reinforcement is not required.

The expression for calculation of ratio x/d for singly reinforced section for HSC has been derived in *Equation* 4-15 as follows

$$\frac{\alpha_{cc}\,\eta\lambda^2}{3}\left(\frac{x}{d}\right)^2 - \frac{\alpha_{cc}\,\eta\lambda}{1.5}\left(\frac{x}{d}\right) + K = 0$$

Substitute α_{cc} = 0.85 as decided in Singapore Annex, λ = 0.775, η= 0.95 for f_{ck} = 60 N/mm^2 shown in Case 1 and K = 0.107 to the above expression we have

$$0.162\left(\frac{x}{d}\right)^2 - 0.417\left(\frac{x}{d}\right) + 0.107 = 0$$

Solving this quadratic equation give x/d = 0.289, x = 0.289×540 = 156 mm

the level arm $z = d$ - 0.5λx = 540 - 0.5×0.775×156 = 479.5mm

$$A_s = \frac{M}{0.87 f_{yk}\,\mathrm{z}} = \frac{560\times 10^6}{0.87\times 500\times 479.5} = 2684\mathrm{mm}^2$$

Provide 6 ϕ25 for A_s, area = 2945 mm^2 > 2684 mm^2

<u>Case 4:</u> Design for 20% moment redistribution, rebar grade B600B to SS 560:2016, using Singapore Annex to EC2

Calculate parameter K

$K = M / bd^2 f_{ck}$ = 560×10^6 / 300×540^2×60 = 0.107

Similar to case 3 above with K = 0.107 < K_{bal} = 0.126, compression reinforcement is not required and x = 156 mm, the level arm z = 479.5mm

$$A_s = \frac{M}{0.87 f_{yk}\,\mathrm{z}} = \frac{560\times 10^6}{0.87\times 600\times 479.5} = 2236\mathrm{mm}^2$$

Provide 5 ϕ25 for A_s, area = 2454 mm^2 > 2236 mm^2

<u>Case 5:</u> Design for 20% moment redistribution, rebar grade B500B to SS 560:2016, using recommended values of EC2

Using recommended values of EC2 we have parameters k_3 = 0.54, k_4 = 1.25(0.6+0.0014/ε_{cu}) = 1.25(0.6+0.0014/0.00288) = 1.357

With 20% redistribution, from *Table 4-5* using recommended values of EC2 we have x_u/d = 0.192 and K_{bal} = 0.074

$K = M / bd^2 f_{ck}$ = 560×10^6 / 300×540^2×60 = 0.107 > K_{bal} thus compression reinforcement is needed.

The neutral axis depth $x = 0.192d$ = 0.192× 540 = 103 mm

Steel compressive strain:

$$\varepsilon_{sc} = \frac{\varepsilon_{cu}(x - d')}{x} = \frac{0.00288(103-50)}{103} = 0.00149 < 0.00217$$

thus $f_{sc} < 0.87\, f_{yk}$

steel compressive stress, $f_{sc} = E_s\varepsilon_{sc}$ = 200 000 $\times$ 0.00149 = 298 N/mm^2

Compression steel area:

$$A_s' = \frac{(K - K_{bal})f_{ck}bd^2}{f_{sc}(d - d')} = \frac{(0.107 - 0.074)\times 60\times 300\times 540^2}{298\times(540 - 50)} = 1175 \text{ mm}^2$$

Try 3ϕ25 for A_s', area = 1473 mm^2

The tension steel area

$$A_s = \frac{K_{bal}f_{ck}bd^2}{0.87f_{yk}z} + A_s'\frac{f_{sc}}{0.87f_{yk}}$$

where $z = d$-0.5λx = 540 - 0.5$\times$0.775$\times$103 = 500 mm

$$A_s = \frac{0.074 \times 60 \times 300 \times 540^2}{0.87 \times 500 \times 500} + 1175\frac{298}{0.87 \times 500} = 2591 \text{ mm}^2$$

Try 4 ϕ25 and 2ϕ20 for A_s, area = 2592 mm^2

Check

$(A_{s,prov}' - A_{s,req}')$ = 1473 - 1175 = 298 mm^2

$(A_{s,prov} - A_{s,req})$ = 2592 - 2591 = 1 mm^2

$(A_{s,prov}' - A_{s,req}')$ > $(A_{s,prov} - A_{s,req})$ => ductility is ensured

<u>Case 6:</u> Design for 20% moment redistribution, rebar grade B600B to SS 560:2016, using recommended values of EC2

Using recommended values of EC2 we have parameters k_3 = 0.54, k_4 = 1.25(0.6+0.0014/ε_{cu}) = 1.25(0.6+0.0014/0.00288) = 1.357

With 20% redistribution, from *Table 4-5* using recommended values of EC2 we have x_u/d = 0.192 and K_{bal} = 0.074

$K = M / bd^2f_{ck}$ = 560×10^6 / 300×540^2×60 = 0.107 > K_{bal} thus compression reinforcement is needed.

The neutral axis depth x = 0.192d = 0.192× 540 = 103 mm

Steel compressive strain:

$$\varepsilon_{sc} = \frac{\varepsilon_{cu}(x - d')}{x} = \frac{0.00288(103 - 50)}{103} = 0.00149 < 0.00261$$

thus $f_{sc} < 0.87 f_{yk}$

steel compressive stress, $f_{sc} = E_s\varepsilon_{sc}$ = 200 000 × 0.00149 = 298 N/mm^2

Compression steel area:

$$A_s' = \frac{(K - K_{bal})f_{ck}bd^2}{f_{sc}(d - d')} = \frac{(0.107 - 0.074)\times 60\times 300\times 540^2}{298\times(540 - 50)} = 1175 \text{ mm}^2$$

Try 3ϕ25 for A_s', area = 1473 mm²

The tension steel area

$$A_s = \frac{K_{bal} f_{ck} b d^2}{0.87 f_{yk} z} + A_s' \frac{f_{sc}}{0.87 f_{yk}}$$

where $z = d$-0.5λx = 540 - 0.5× 0.775× 103 = 500 mm

$$A_s = \frac{0.074 \times 60 \times 300 \times 540^2}{0.87 \times 600 \times 500} + 1175 \frac{298}{0.87 \times 600} = 2159 \text{ mm}^2$$

Try 5 ϕ25 for A_s, area = 2454 mm²

Check

$(A'_{s,prov} - A'_{s,req})$ = 1473 - 1175 = 298 mm²

$(A_{s,prov} - A_{s,req})$ = 2454 - 2159 = 295 mm²

$(A'_{s,prov} - A'_{s,req})$ > $(A_{s,prov} - A_{s,req})$ => ductility is ensured

Table 2 Summary of required reinforcement area for the six cases of example 3-2

Rebar	No moment redistribution		20% Moment redistribution			
			Singapore Annex		Recommended value EC2	
	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
	f_{yk} = 500 N/mm2	f_{yk} = 600 N/mm2	f_{yk} = 500 N/mm2	f_{yk} = 600 N/mm2	f_{yk} = 500 N/mm2	f_{yk} = 600 N/mm2
$A'_{s,req}$	180	180	0	0	1175	1175
$A_{s,req}$	3459	2866	2684	2236	2591	2159
$A'_{s,prov}$	628	628	0	0	1473	1473
$A_{s,prov}$	3572	2945	2945	2454	2592	2454

Comment: As mentioned earlier in this section, for HSC, using the EC2 recommended parameters to design for moment redistribution would result in higher ductility or rotational capacity compared to those stipulated by UK and Singapore Annex, as a result the former requires more compression reinforcement than the latter as can be seen from Table 2 for the same percentage of moment redistribution. The total reinforcement area after moment redistribution is even higher than before redistribution in case of using EC2 recommended parameters. With the same parameters, a section using higher strength reinforcement requires less tension steel than the one using lower strength reinforcement but the required amount of compression steel remains unchanged. It is necessary to check that the extra amount of reinforcement provided for compression reinforcement compared to the required area shall not be less than that for tension reinforcement to ensure the ductility requirement.

4. Prediction of confinement of concrete using the Eurocode model

Example 4.1. Designing confinement reinforcement required with f_{ck} = 40 MPa

For a 500mm x 500mm square concrete column with f_{ck} = 40 MPa to achieve an increment of strain at peak stress of concrete by 0.0005, i.e $\varepsilon_{c2,c}$ - ε_{c2} = 0.5‰, for two cases using Grade B500 and Grade B600 reinforcement. Assuming concrete cover to the link is 30mm and column is reinforced with 8H25 as shown in Figure 6.

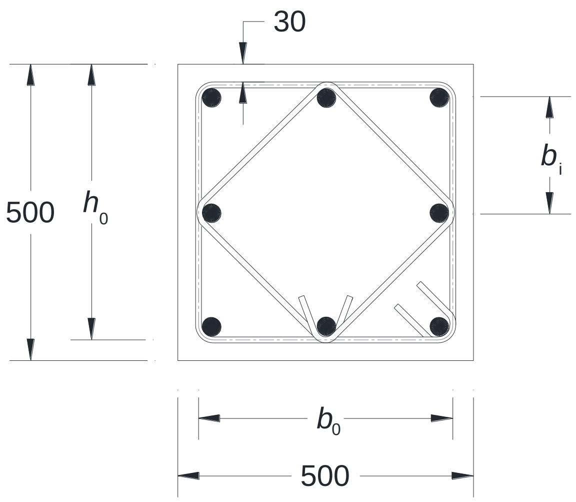


Figure 6 Illustration of Example 4.1

Case 1. Using Grade B500 (f_{yw} = 500 MPa) for transverse reinforcement.

Use 10mm diameter reinforcement for the perimeter link and diagonal hoop (A_s = 78.54 mm^2). The dimension of core concrete will be

$b_0 = h_0$ = 500-2×30-10 = 430 mm

The distance between consecutive engaged bars b_i will be the same around 4 sides and can be computed as

b_i = (430 - 10 - 25)/2 = 197.5 mm

the ratio of volume of transverse reinforcement to the volume of core concrete will be estimated as

$\rho_{sx} = 4b_0\,(1+\frac{\sqrt{2}}{2})A_s\,/\,(b_0^2 s) = 1.247/s$

where s is the spacing of transverse reinforcement in mm, A_s is cross section area of transverse reinforcement (A_s = 78.54 mm^2)

From Equation 4-31 we have

$$\alpha = \left(1 - \frac{s}{2b_0}\right)^2 \left(1 - \frac{\sum b_i^2}{6b_0^2}\right) = 0.719\left(1 - \frac{s}{860}\right)^2$$

First, assuming $\sigma_2 < 0.05 f_{ck}$, from *Equation 4-28* and *Equation 4-29* we have

$$\varepsilon_{c2,c} = \varepsilon_{c2}(1.000 + 5.0\ \sigma_2/f_{ck})^2$$

the requirement is $\varepsilon_{c2,c} - \varepsilon_{c2} = 0.5‰$ thus we can write

$$\varepsilon_{c2}\ [(1.000 + 5.0\ \sigma_2/f_{ck})^2 - 1] = 0.5‰$$

From Table 3.1 of EC2, with f_{ck} = 40 MPa we have ε_{c2} = 2‰, hence

$$2‰[(1.000 + 5.0\ \sigma_2/40)^2 - 1] = 0.5‰$$

Solving this quadratic equation gives $\sigma_2 = \sqrt{80}-8 = 0.944$ MPa.

Check against assumption σ_2 = 0.944 MPa < 0.05 f_{ck} = 2.0 MPa -> Satisfied

Replacing the parameters σ_2, ρ_{sx} , α and f_{yw} = 500 MPa into Equation 4-30 we have

$$0.944 = 0.719\left(1-\frac{s}{860}\right)^2\frac{1.247}{s}(500)$$

This is a quadratic equation that gives s = 243 mm and a volumetric ratio $\rho_{sx} = 1.247/s$ = 0.51%.

Case 2. Using Grade B600 (f_{yw} = 600 MPa) for transverse reinforcement.

The parameters σ_2, ρ_{sx} , α are similar to Case 1, thus we have equation for spacing s as

$$0.944 = 0.719\left(1-\frac{s}{860}\right)^2\frac{1.247}{s}(600)$$

solving this quadratic equation gives s = 269 mm and a volumetric ratio $\rho_{sx} = 1.247/s$ = 0.46%.

Example 4.2. Designing confinement reinforcement required with f_{ck} = 80 MPa

Similar to example 4.1 but the column uses high strength concrete f_{ck} = 80 MPa.

From Table 3.1 of EC2, for f_{ck} = 80 MPa we get ε_{c2} = 2.5‰, thus the requirement for confinement can be written as

$$2.5‰\ [(1.0 + 5.0\ \sigma_2/80)^2 - 1] = 0.5‰$$

Solving this quadratic equation gives σ_2 = 1.53 MPa < 0.05 f_{ck} = 4 MPa

Substitute this effective lateral compressive stress for Case 1 and Case 2 of example 4.1, the required spacing of transverse reinforcement for Case 1 would be s = 182 mm (ρ_{sx} = 0.69%) and for Case 2 would be s = 204mm (ρ_{sx} = 0.61%).

Example 4.3. Designing confinement reinforcement as circular hoops, f_{ck} = 40 MPa

For a 500mm diameter circular concrete column with f_{ck} = 40 MPa to achieve an increment of strain at peak stress of concrete by 0.0005, i.e $\varepsilon_{c2,c} - \varepsilon_{c2} = 0.5‰$, for two cases using Grade B500 and Grade B600 reinforcement. Assuming concrete cover to the link is 30mm and column is reinforced with 8H25 as shown in Figure 7.

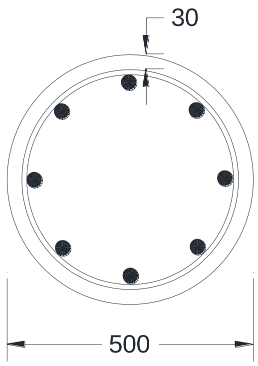


Figure 7 Illustration of Example 4.3

Case 1. Using Grade B500 (f_{yw} = 500 MPa) for transverse reinforcement.

Use 10mm diameter reinforcement for the circular hoop (A_s = 78.54 mm^2). The diameter of core concrete will be

D_0 = 500-2×30-10 = 430 mm

the ratio of volume of transverse reinforcement to the volume of core concrete will be estimated as

$\rho_{sx} = \pi D_o A_s / (\pi D_0^2 s/4) = 0.73/s$

where s is the spacing of transverse reinforcement in mm

From Equation 4-32 we have

$$\alpha = \left(1 - \frac{s}{2D_0}\right)^2 = \left(1 - \frac{s}{860}\right)^2$$

Similar to example 4.1, with f_{ck} = 40 MPa, ε_{c2} = 2‰ we need an effective lateral compressive stress σ_2 = 0.944 MPa, replacing the parameters σ_2, ρ_{sx} , α and f_{yw} = 500 MPa to Equation 4-30 we have

$$0.944 = \left(1 - \frac{s}{860}\right)^2 \frac{0.73}{s}(500)$$

This is a quadratic equation that gives s = 216 mm and a volumetric ratio $\rho_{sx} = 0.73/s$ = 0.34%.

Case 2. Using Grade B600 (f_{yw} = 600 MPa) for transverse reinforcement.

the parameters σ_2, ρ_{sx} , α are similar to Case 1, thus we have equation for spacing s as

$$0.944 = \left(1 - \frac{s}{860}\right)^2 \frac{0.73}{s}(600)$$

solving this quadratic equation gives s = 240 mm and a volumetric ratio $\rho_{sx} = 0.73/s$ = 0.30%

Example 4.4. Designing confinement reinforcement as circular hoops, f_{ck} = 80 MPa

Similar to example 4.3 but the column use high strength concrete f_{ck} = 80 MPa.

The effective lateral compressive stress is already determined in Example 4.2

σ_2 = 1.53 MPa

Substitute this effective lateral compressive stress for Case 1 and Case 2 of Example 4.3, the required spacing of transverse reinforcement for Case 1 would be s = 159 mm (ρ_{sx} = 0.46%) and for Case 2 would be s = 179mm (ρ_{sx} = 0.41%).

5. Estimation of confining reinforcement amount to utilize the high strength reinforcement based on confinement mechanism given in EC2

Example 5.1: f_{ck} = 40 MPa f_{yk} = 600 MPa

Consider columns with square section subjected to concentric loading, given nominal concrete cover to the link is 30mm, diameter of the link is 10mm and the yield strength of the link f_{yw} = 500 MPa, concrete strength f_{ck} = 40 MPa, assuming yield strength of longitudinal reinforcement is f_{yk} = 600 MPa and the ratio ρ_s = 1%, assuming the link include a perimeter hoop and n internal ties in each direction.

Fixed parameters: c = 30mm, f_{ck} = 40 MPa, ds = 10mm, f_{yw} = 500 MPa, f_{yk} = 600 MPa (ε_{ykd} = 2.61‰), ρ_s = 1%

The required amount of transverse steel ratios for various column size to utilize design yield strength of reinforcement can be computed from Equation 4-37 to Equation 4-41, the results are as follows:

b (mm)	b_0 (mm)	f_{ck} (MPa)	ε_{c2} (‰)	required $f_{ck,c}$ (MPa)	$\varepsilon_{c2,c}$ (‰)	σ_2 (MPa)	n	s (mm)	ρ_{sh}
300	230	40	2.00	64.82	5.25	12.85	1	44	5.35%
400	330	40	2.00	55.85	3.90	6.06	1	64	2.54%
500	430	40	2.00	51.33	3.29	3.25	2	127	0.98%
600	530	40	2.00	48.62	2.95	2.10	2	159	0.64%
700	630	40	2.00	46.81	2.74	1.60	3	221	0.38%
800	730	40	2.00	45.68	2.61	1.30	4	281	0.26%
900	830	40	2.00	45.68	2.61	1.30	4	267	0.24%
1000	930	40	2.00	45.68	2.61	1.30	5	289	0.20%
1200	1130	40	2.00	45.68	2.61	1.30	6	293	0.16%

It can be noted that the lost of concrete cover has lesser effect to columns with bigger size. For small size such as 300x300mm the required volumetric ratios of transverse reinforcement of

5.35% is too high and not practical. For a size greater than 700x700, the required confined concrete strength is governed by the required design yield strength of rebar $\varepsilon_{c2,c} = \varepsilon_{ykd}$.

In case of High strength concrete, similar to the above example but using f_{ck} = 80 MPa we can compute the required amount of transverse steel ratios as follows:

b (mm)	b_0 (mm)	f_{ck} (MPa)	ε_{c2} (‰)	required $f_{ck,c}$ (MPa)	$\varepsilon_{c2,c}$ (‰)	σ_2 (MPa)	n	s (mm)	ρ_{sh}
300	230	80	2.52	136.51	7.32	31.744	1	20	11.83%
400	330	80	2.52	117.61	5.44	16.235	1	27	6.03%
500	430	80	2.52	108.10	4.59	9.784	2	51	2.43%
600	530	80	2.52	102.39	4.12	6.345	2	64	1.58%
700	630	80	2.52	98.59	3.82	4.234	3	103	0.82%
800	730	80	2.52	95.87	3.61	3.805	4	123	0.60%
900	830	80	2.52	93.84	3.46	3.247	4	129	0.50%
1000	930	80	2.52	92.26	3.35	2.827	5	156	0.37%
1200	1130	80	2.52	89.96	3.18	2.240	6	189	0.25%

The required confined concrete strength is governed by the capacity of column at maximum unconfined concrete strength with $\varepsilon_{c2,c}$ always higher than ε_{ykd} for f_y = 600 MPa. For those columns that have size smaller than 500x500 the required volumetric ratios of transverse reinforcement of 6.03% or more are too high and not practical.

Example 5.2 : f_{ck} = 80 MPa f_{yk} = 600 MPa

Consider columns with circular section confined by circular hoop subjected to concentric loading, given nominal concrete cover to the link is 30mm, diameter of the link is 10mm and the yield strength of the link f_{yw} = 500 MPa, concrete strength f_{ck} = 80 MPa, assuming yield strength of longitudinal reinforcement is f_y = 600 MPa and the ratio ρ_s = 1%, assuming the link include a perimeter hoop and n internal ties in each direction

Fixed parameters: c = 30mm, f_{ck} = 80 MPa, d_s = 10mm, f_{yw} = 500 MPa, f_y = 600 MPa (ε_y = 2.61‰), ρ_s = 1%

D (mm)	D_0 (mm)	f_{ck} (MPa)	ε_{c2} (‰)	required $f_{ck,c}$ (MPa)	$\varepsilon_{c2,c}$ (‰)	σ_2 (MPa)	s (mm)	ρ_{sh}
300	230	80	2.52	136.51	7.32	31.744	20	11.83%
400	330	80	2.52	117.61	5.44	16.235	27	6.03%
500	430	80	2.52	108.10	4.59	9.784	51	2.43%
600	530	80	2.52	102.39	4.12	6.345	64	1.58%
700	630	80	2.52	98.59	3.82	4.234	103	0.82%
800	730	80	2.52	95.87	3.61	3.805	123	0.60%
900	830	80	2.52	93.84	3.46	3.247	129	0.50%
1000	930	80	2.52	92.26	3.35	2.827	156	0.37%
1200	1130	80	2.52	89.96	3.18	2.240	189	0.25%

The required amount of transverse steel ratios for various column diameters can be computed from *Equation 4-42* to *Equation 4-45*, the results are as follows:

It can also be seen that for those columns that have size less than D500 the required volumetric ratios of transverse reinforcement of 6.03% or more is too high and not practical.

6. Effect of creep and shrinkage on load redistribution in columns

Example 6.1: f_y = 600 MPa, concrete grade C30/37 using cement class N

Given a 500x500 RC column, reinforced with 12 nos of H20 rebar (A_s =3770mm^2, ρ_s = 1.5%) with f_y = 600 MPa and E_s = 200 GPa, concrete grade C30/37 using cement class N, subjected to concentrate sustained loading equal to 50% of the design ultimate axial capacity at 28 days after casting of concrete. The concrete were cured for 5 days and the working environment after curing has the Relative Humidity RH= 70%. Determine the total strain and the redistribution of stress from concrete to the reinforcement at the age of two years (730 days), considering additional strain from shrinkage of concrete.

For concrete grade C30/37 the strain at maximum stress may be assumed to follow Table 3.1 and Figure 3.3 of EC2 which give ε_{c2} = 0.002 less than strain at design yield strength of rebar, thus the design ultimate axial capacity of the column is

$$P_{uD} = 0.85 f_{ck} A_c / \gamma_c + \varepsilon_{c2} E_s A_s$$

$$P_{uD} = [0.85\times30\ (500\times500 - 3770)/1.5 + 0.002\times2\times10^5\times3770\ ]/1000 = 5694 \text{ kN}$$

The sustained loading $P = 0.5\ P_{uD} = 0.5\times5694 = 2847$ kN

- Effect of creep:

Since the column is loaded at 28 days, the concrete strength at time of loading is equal to f_{ck} (t_0) = 30 MPa and the mean compressive strength of concrete $f_{cm}(t_0)$ = 38 MPa

- Concrete strength coefficients:

$\alpha_1 = (35/f_{cm})^{0.7} = 0.944$; $\alpha_2 = (35/f_{cm})^{0.2} = 0.984$; $\alpha_3 = (35/f_{cm})^{0.5} = 0.960$

- Humidity factor:

Notional size of column: $h_0 = 2A_c/u$ = 250 mm

since f_{cm} = 38 > 35, $\varphi_{RH} = \left[1 + \frac{1+RH/100}{0.1\sqrt[3]{h_0}}\alpha_1\right]\alpha_2$ =1.426

- Concrete strength factor:

$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}}$ = 2.725

- Concrete age factor:

$\beta(t_0) = \frac{1}{0.1+t_0^{0.20}}$ = 0.488

- Notional creep coefficient:

$\varphi_0 = \varphi_{RH}.\beta(f_{cm}).\ \beta(t_0) = 1.426\times2.725\times0.488 = 1.898$

- Relative humidity coefficient:

for f_{cm}> 35 MPa we have $\beta_H = 1.5\ [1+(0.012\ RH)^{18}]h_0 + 250\ \alpha_3 = 631$

$\beta_H \leq 1500\alpha_3$ = 1439.57 , OK

- Creep development coefficient:

$\beta_c(t,t_0) = \left[\frac{t-t_0}{\beta_H+t-t_0}\right]^{0.3}$ = 0.825

The creep coefficient for 2 years age (t=730 days) and loading at 28 days age (t_0 = 28) is:

$\varphi\ (t,t_0) = \varphi_0.\beta_c(t,t_0) = 1.898\times0.825 = 1.566$

For iteration calculation of creep strain, the first step is to calculate the initial parameters as follows:

- Tangent modulus of concrete: $E_c = 1.05\ E_{cm} = 1.05\times22\ (f_{cm}/10)^{0.3}$ = 34.48 GPa

- Initial modular ratio: $\alpha_e = E_s / E_c = 200/34.48 = 5.80$

- Concrete stress at the time of applying load t_0 (at 28 days):

$\sigma_0 = \frac{N}{A_c+\alpha_e A_s}$ = 10.62 MPa

since $\sigma_0 < 0.45\ f_{ck}(t_0) = 0.45\times30 = 13.5$ MPa creep non-linearity need not be considered.

- Rebar stress at the time of applying load t_0 :

$\sigma_{s0} = \alpha_e\sigma_0 = 5.80\times10.62 = 61.60$ MPa

- Short-term elastic strain of concrete

$$\varepsilon_0 = \sigma_0 / E_c = 10.62 / 34.48\times10^3 = 0.000308$$

After two years, at first iteration assuming the concrete is subject to the initial stress σ_0 then the creep strain can be estimated as

$$\varepsilon_{cc1}(t,t_0) = \varphi(t,t_0).(\sigma_0/E_c) = 1.566\times0.000308 = 0.000482$$

and the total strain will be

$$\varepsilon_{c1} = \varepsilon_0 + \varepsilon_{cc1}(t,t_0) = 0.000308+0.000482 = 0.00079$$

However due to the increase of strain, the effective modulus of concrete will be reduced to

$$E_{ce1} = \sigma_0 / \varepsilon_{c1} = 10.62 / 0.00079 = 13437 \text{ MPa} = 13.437 \text{ GPa}$$

the effective modular ratio will be increased to

$$\alpha'_{e1} = E_s / E_{ce1} = 200/13.437 = 14.88$$

consequently the concrete stress will be reduced to

$$\sigma_{c1} = \frac{N}{A_c+\alpha'_e A_s} = 9.42 \text{ MPa}$$

As $\sigma_{c1}<\sigma_0$ we need to repeat the iteration by substitute σ_{c1} to σ_0 in the first step and repeat this process which yields the following result:

step 2:

$\varepsilon_{cc2} = 0.000428$; $\varepsilon_{c2} = 0.000736$; $E_{ce2} = 12.80$ GPa; $\alpha'_{e2} = 15.62$; $\sigma_{c2} = 9.33$ MPa

step 3:

$\varepsilon_{cc3} = 0.000424$; $\varepsilon_{c3} = 0.000732$; $E_{ce3} = 12.75$ GPa; $\alpha'_{e3} = 15.69$; $\sigma_{c3} = 9.32$ MPa

The error between results from step 3 and step 2 is less than 1% so we can stop at step 3. From this analysis, the additional compressive strain that the rebar has to take from the creep effect is 0.000424 after two years.

- ***Effect of shrinkage***

Since the column is under compression, the shrinkage of concrete is only restrained internally by the rebar.

The nominal unrestrained drying shrinkage value $\varepsilon_{cd,0}$ can be computed from Appendix B or interpolated from Table 3.2 of SS EN 1992-1-1 for RH = 70, cement type N and f_{ck} = 30 MPa, which gives $\varepsilon_{cd,0}$ = 0.00036. Interpolated from Table 3.3 of the code with h_0 = 250 as computed earlier gives k_h = 0.80.

The age of the concrete at the beginning of drying shrinkage is taken at the end of curing time, thus t_s = 5 days, at the moment considered, t = 730 days, we have

$$\beta_{sd}(t,t_s)=\frac{(t-t_s)}{(t-t_s)+0.04\sqrt{h_0^3}}\ = 0.82$$

The total drying shrinkage value at 730 days can be determined as

$\varepsilon_{cd}\ (t) = \beta_{ds}(t,\ t_s)\cdot k_h\cdot \varepsilon_{cd,0} = 0.82\times 0.80\times 0.00036 = 0.000238$

The autogenous shrinkage strain can be estimated at time t=730 days as

$\varepsilon_{ca}(\infty) = 2.5\ (f_{ck} - 10)\ 10^{-6} = 2.5\ (30 - 10)\ 10^{-6} = 0.00005$

$\beta_{as}(t) = 1 - \exp\ (-\ 0.2t^{\ 0.5}) = 1 - \exp\ (-\ 0.2\times(730)^{0.5}) = 0.996$

$\varepsilon_{ca}\ (t) = \beta_{as}(t)\ \varepsilon_{ca}(\infty) = 0.00005$

Total free shrinkage strain of concrete at time t=730 days is

$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca} = 0.000238 + 0.00005 = 0.000288$

For concrete f_{ck} = 30 MPa, the modulus of elasticity of concrete is E_{cm} = 32.84 GPa, for steel E_s = 200 GPa

thus the ratio $\alpha_e = E_s\,/\,E_{cm}$ = 200 /33 = 6.09

The additional compressive strain in the rebar can be estimated as

$$\varepsilon_{sc}=\frac{\varepsilon_{cs}}{1+\frac{\alpha_e A_s}{A_c}} = 0.000263$$

Summary: The combined effects of creep and shrinkage produce an additional compressive strain in the reinforcement of 0.000424 + 0.000263 = 0.000687. This strain combined with the strain due to external force at ultimate limit state of 0.002 given in EC2, will exceed the design yield strength of the B600 rebar.

Example 6.2 : f_y = 600 MPa , concrete grade C70/85 using cement class N

Given a 500x500 RC column, reinforced with 12 nos of H20 rebar (A_s =3770mm^2, ρ_s = 1.5%) with f_y = 600 MPa and E_s = 200 GPa, concrete class C70/85 using cement class N, subjected to concentrate sustained loading equal to 50% of the design ultimate axial capacity at 28 days after casting of concrete. The concrete were cured for 5 days and the working environment after curing has the Relative Humidity *RH*= 70%. Determine the total strain and the redistribution of stress from concrete to the reinforcement at the age of two years (730 days), considering additional strain from shrinkage of concrete.

For concrete class C70/85 the strain at maximum stress is ε_{c2} = 0.0024 less than the strain at design yield strength of rebar (0.0026), thus the design ultimate axial capacity of the column is

$P_{uD} = 0.85 f_{ck} A_c\,/\,\gamma_c + \varepsilon_{c2} E_s A_s$

P_{uD} = [0.85×70 (500×500 - 5890)/1.5 + 0.0024×2×10^5×3770]/1000 = 11577 kN

The sustained loading $P = 0.5\ P_{uD}$ = 0.5×11577 = 5788 kN

- Effect of creep:

Since the column is loaded at 28 days, the concrete strength at time of loading is equal to $f_{ck}\ (t_0)$ = 70 MPa and the mean compressive strength of concrete $f_{cm}(t_0)$ = 78 MPa

- Concrete strength coefficients:

$\alpha_1 = (35/f_{cm})^{0.7} = 0.571$; $\alpha_2 = (35/f_{cm})^{0.2} = 0.852$; $\alpha_3 = (35/f_{cm})^{0.5} = 0.670$

- Humidity factor:

Notional size of column: $h_0 = 2A_c/u$ = 250 mm

since f_{cm} = 78 > 35, $\varphi_{RH} = \left[1 + \frac{1+RH/100}{0.1\sqrt[3]{h_0}}\alpha_1\right]\alpha_2$ =1.083

- Concrete strength factor:

$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}} = 1.902$

- Concrete age factor:

$\beta(t_0) = \frac{1}{0.1+t_0^{0.20}} = 0.488$

- Notional creep coefficient:

$\varphi_0 = \varphi_{RH}.\beta(f_{cm}).\ \beta(t_0)$ = 1.083×1.902×0.488 = 1.007

- Relative humidity coefficient:

for f_{cm}> 35 MPa we have $\beta_H = 1.5\ [1+(0.012\ RH)^{18}]h_0 + 250\ \alpha_3 = 558.72$

$\beta_H \le 1500\alpha_3$ = 1004.8 , OK

- Creep development coefficient:

$\beta_c(t,t_0) = \left[\frac{t-t_0}{\beta_H+t-t_0}\right]^{0.3} = 0.839$

The creep coefficient for 2 years age (t=730 days) and loading at 28 days age (t_0 = 28) is:

$\varphi\ (t,t_0) = \varphi_0.\beta_c(t,t_0)$ = 1.007×0.839 = 0.844

For iteration calculation of creep strain, the first step is to calculate the initial parameters as follows:

- Tangent modulus of concrete: $E_c = 1.05\ E_{cm} = 1.05\times22\ (f_{cm}/10)^{0.3}$ = 42.78 GPa

- Initial modular ratio: $\alpha_e = E_s / E_c$ = 200/42.78 = 4.68

- Concrete stress at the time of applying load t_0 (at 28 days):

$$\sigma_0 = \frac{N}{A_c+\alpha_e A_s} = 21.94 \text{ MPa}$$

since $\sigma_0 < 0.45\, f_{ck}(t_0) = 0.45\times70 = 31.5$ MPa creep non-linearity need not be considered.

- Rebar stress at the time of applying load t_0 :

$$\sigma_{s0} = \alpha_e\sigma_0 = 4.68\times21.94 = 102.56 \text{ MPa}$$

- Short-term elastic strain of concrete

$$\varepsilon_0 = \sigma_0 / E_c = 21.94 / 42.78\times10^3 = 0.000513$$

After two years, at first iteration assuming the concrete is subject to the initial stress σ_0 then the creep strain can be estimated as

$$\varepsilon_{cc1}(t,t_0) = \varphi(t,t_0).(\sigma_0/E_c) = 0.844\times0.000513 = 0.000433$$

and the total strain will be

$$\varepsilon_{c1} = \varepsilon_0 + \varepsilon_{cc1}(t,t_0) = 0.000513+0.000433 = 0.000946$$

However due to the increase of strain, the effective modulus of concrete will be reduced to

$$E_{ce1} = \sigma_0 / \varepsilon_{c1} = 21.94 / 0.000946 = 23.193 \text{ MPa} = 23.193 \text{ GPa}$$

the effective modular ratio will be increased to

$$\alpha'_{e1} = E_s / E_{ce1} = 200/23.193 = 8.62$$

consequently the concrete stress will be reduced to

$$\sigma_{c1} = \frac{N}{A_c+\alpha'_e A_s} = 20.77 \text{ MPa}$$

As $\sigma_{c1}<\sigma_0$ we need to repeat the iteration by substitute σ_{c1} to σ_0 in the first step and repeat this process which yields the following result:

step 2:

$\varepsilon_{cc2} = 0.00041$; $\varepsilon_{c2} = 0.000923$; $E_{ce2} = 22.507$ GPa; $\alpha'_{e2} = 8.89$; $\sigma_{c2} = 20.69$ MPa

step 3:

$\varepsilon_{cc3} = 0.000408$; $\varepsilon_{c3} = 0.000921$; $E_{ce3} = 22.453$ GPa; $\alpha'_{e3} = 8.907$; $\sigma_{c3} = 20.69$ MPa

The error between results from step 3 and step 2 is less than 1% so we can stop at step 3. From this analysis, the additional compressive strain that the rebar has to take from the creep effect is 0.000408 after two years.

- ***Effect of shrinkage***

Since the column is under compression, the shrinkage of concrete is only restrained internally by the rebar.

The nominal unrestrained drying shrinkage value $\varepsilon_{cd,0}$ can be computed from Appendix B or interpolated from Table 3.2 of SS EN 1992-1-1 for RH = 70%, cement type N and f_{ck} = 70 MPa, which gives $\varepsilon_{cd,0}$ = 0.000224. Interpolated from Table 3.3 of the code with h_0 = 250 as computed earlier gives k_h = 0.80.

The age of the concrete at the beginning of drying shrinkage is taken at the end of curing time, thus t_s = 5 days, at the moment considered, t = 730 days, we have

$$\beta_{sd}(t, t_s) = \frac{(t-t_s)}{(t-t_s)+0.04\sqrt{h_0^3}} = 0.82$$

The total drying shrinkage value at 730 days can be determined as

$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0} = 0.82 \times 0.80 \times 0.000224 = 0.000147$

The autogenous shrinkage strain can be estimated at time t=730 days as

$\varepsilon_{ca}(\infty) = 2.5\,(f_{ck} - 10)\,10^{-6} = 2.5\,(70 - 10)\,10^{-6} = 0.000150$

$\beta_{as}(t) = 1 - \exp(-0.2t^{0.5}) = 1 - \exp(-0.2 \times (730)^{0.5}) = 0.996$

$\varepsilon_{ca}(t) = \beta_{as}(t)\,\varepsilon_{ca}(\infty) = 0.000149$

Total free shrinkage strain of concrete at time t=730 days is

$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca} = 0.000147 + 0.000149 = 0.000296$

For concrete f_{ck} = 70 MPa, the modulus of elasticity of concrete is E_{cm} = 40.74 GPa, for steel E_s = 200 GPa

thus the ratio $\alpha_e = E_s / E_{cm} = 200 / 40.74 = 4.91$

The additional compressive strain in the rebar from shrinkage effect can be estimated as

$$\varepsilon_{sc} = \frac{\varepsilon_{cs}}{1+\frac{\alpha_e A_s}{A_c}} = 0.000276$$

Summary: The combined effects of creep and shrinkage produce an additional compressive strain in the reinforcement of 0.000408 + 0.000276 = 0.000684. This strain combined with the strain due to external force at ultimate limit state of 0.0024 given in EC2, will exceed the design yield strength of the B600.

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